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TEXTURAL IMAGES REPRESENTATION USING FRACTAL GEOMETRY MEASURES

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ABSTRACT

Fractal functions are good choice for modeling natural textured surfaces and they have long been recognized as important features in classifying images because the fractal dimension for a surface correlates nearly perfectly with the perception of roughness in many situations. The use of new set of fractal features to identify visual texture is explored in this paper. We have adopted a set of two fractal features (i.e. fractal dimension and lacunarity) for describing different visual textures. A modified differential box counting approach for estimating these two fractal features from image surfaces is proposed. Also, a focus on these two fractal features (parameters), with their accuracy and robustness were evaluated. The problem of textural images description was presented. The research conducted a testing procedure to evaluate the degree of sensitivity against the textural attributes (i.e. softness, feature size and power). The results have shown satisfactory results, which can justify the usage of fractal features as textural discriminating criteria.

Keywords- *Fractal, Textural, Lacunarity etc*

I. INTRODUCTION

The science of ***fractal geometry*** is modern invention discovered by the polish-born French mathematician ***Benoit B. Mandelbrot*** in the 1970s. It provides both a description and a mathematical model for many of the seemingly complex forms and patterns in nature and sciences.

Within the last 15-30 years, ***fractal geometry*** and its concepts have become central tools in most of the natural sciences: physics, chemistry, biology, meteorology, and material science, at the same time, ***fractals*** are of interest to computer graphic designers and film-makers for their ability to create new and exciting shapes and artificial but are very close to realistic worlds.

Fractal analysis, the study of complicated phenomena manifesting self-similarity at many scales, is suited to the description of the forms and sizes of the very complicated, revealing regions homogeneously classified pixels with quite convolutes perimeters.

Computer graphic has played an essential role in the development and rapid acceptance of ***fractals*** as a valid new discipline. Conversely, ***fractal geometry*** now plays an important role in the rendering, modeling and three-dimensional animation of natural phenomena and fantastic shapes in computer graphics.

The using of ***fractal geometry*** concepts in the fields of computer vision and digital image processing is considered to be a modern fascinating and challenging field. It is fascinating because of the possibilities it offers and it is challenging because, it is often to "understand" and "improve" unusual image aspects and properties, and this has proved to be no simple task.

For the past 15 years, computer vision and digital image processing through ***fractal geometry*** have grown into mature discipline. In addition, its techniques in the

Image analyses are a rapidly growing field and this is due, at least in part to recent advances in desktops and personal computers.

The discovery of the ***fractal geometry*** expands the applications of the image processing to many problems of:

a) Representing natural shapes such as mountains, trees, and clouds, and

b) Computing their description from image data.

c) The theory of curves and surfaces have been developed in two, three or higher dimensions for about 200 years now. Such like shapes that globally may have very complicated external structures, but if one looks to them in small scales, they are just straight lines or planes. The discipline that deals with such like objects is **"Differential Geometry.** It is considered to be one of the most evolved and fascinating subjects in mathematics.

On the other hand natural shapes such as coastlines, mountains, trees clouds, leaves, feathers, flowers, carpets, and much else besides are not easily described by the above discipline. Nevertheless, they often posses a remarkable simplifying invariance under change of magnification. It is only recently; the scientists have been able to describe the natural shapes, because it is recently that branch of mathematics capable to describe such like complex objects were developed.

Benoit B. Mandelbrot suggested the existence of geometrical entities near to the "***geometry of nature***", thus, he coined them "***fractals***" in 1970. ***Fractal feature*** is just the opposite of smoothness. While the smooth object do not yield any more details on smaller scales a ***fractal*** possesses infinite details at all scales no matter how small they are.

Fractal, in mathematics, a geometric shape that is complex and detailed in structure at any level of magnification. Often ***fractals*** are self-similar, that is, they have the property that each small portion of the ***fractal*** can be viewed as a reduced-scale replica of the whole.

The fascination that surrounds ***fractals*** has two roots, they are:

1. ***Fractals*** are very suitable to simulate many natural phenomena, and

2. ***Fractals*** are simple to generate on computers.

The property of self-similarity or "*scale–invariance*" is one of the central concepts of ***fractals***. A small portion of the figure resembles some larger part, either exactly or very closely. This characteristic was discovered in a wide variety of natural and Man-made phenomenon and traditional Euclidean shapes [Barnsley et al. 88]. Irregular sets provide a much better representation of many natural phenomena than do the figures of classical geometry [Barnsley and Hurd 93]. ***Fractal geometry*** provides a general framework for the study of such irregular sets. Table (2.1) summarizes the major differences between ***fractals*** and traditional Euclidean shapes.

II. FRACTAL (SIMILARITY) DIMENSION

Fractals have infinite details at all scales, so we cannot make a complete computation of a ***fractal*** and some approximations of ***fractals*** down to some finite precision have to suffice.

An object normally considered as a line or line segment, we give it dimension (**D=1**), or a plane or half plane or disk, we give it dimension (**D=2**), or surface or space or half space or ball, we give it dimension (**D=3**), we call **D** the *Topological Dimension*.

Now let's develop a second measure of an object's dimension. If one takes a line of length (**L**) and dividing it into (**N**) identical pieces each of length (**l =L/N**). The pieces each look like the original, only scales by ratio (**r=1/N**) from the whole. Similarly, a two-dimensional objects; such as a square area in the plane, can be divided into "**N**" self–similar parts each of which is scaled down by a factor ($r=1/\sqrt{N}$). A three-dimensional object like a solid cube may be divided into "**N**" little cubes each of which is scaled down by ratio ($r=1/\sqrt[3]{N}$) [Barnsley et al. 88].

With self-similarity the generalization to ***fractal dimension*** is straightforward. A D-dimensional self-similar object can be divided into "**N**" smaller copies of itself each of which is scaled down by a factor "**r**" where

$$r = \frac{1}{\sqrt[D]{N}} ,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Or

$$N = \frac{1}{r^D} ,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Conversely, given a self- similar objects of "**N**" parts scaled by a ratio "**r**" from the whole, its ***fractal*** or ***similarity dimension*** is given by:

$$D = \frac{\log(N)}{\log(\frac{1}{r})}, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The fractal dimension, unlike the more familiarization of topological Euclidean dimension, need not be an integer.

Because it is unreasonable to expect a physical surface to be ***fractal*** over all scales, the only physically reasonable definition of a "***fractal surface***" is a surface that may be accurately approximated by a single ***fractal dimension*** over a range of scales [Keller et al. 87]. We shall say, therefore, that a surface is ***fractal*** if its ***fractal dimension*** is stable over a wide range of scale, the implication being that it can be accurately approximated over that range of scales by a single ***fractal*** function.

There are various numbers, associated with ***fractals***, which can be used to compare them. They are generally referred to as "***fractal dimensions***"[Barnsley 93]. They are attempts to quantify a subjective feeling that we have about how densely the ***fractal*** occupies the metric space in which it lies. The most important thing about ***fractal dimensions*** is that they provide an objective means for comparing ***fractals***.

Fractal dimensions are important because they can be defined in connection with real-world data, and they can be measured approximately by means of experiments.

When a natural object or phenomenon is modeled mathematically by using a ***fractal dimension***, it is recognized that the object or phenomenon does not often have significant *uncorrelation*. In the ***fractal theory***, a ***fractal dimension*** is defined as a determined value (*dimension*) which is independent of the scale of its covering. In practice, however, the dimension often fluctuates depending on samples used (even with same object or phenomena) and its scale. In published works of image analyses and computer vision applications using ***fractal dimensions***, it has been assumed that ***fractal dimensions*** are determined and independent of scales, ignoring the practical experience [Clarke 86].

Fractal dimension is a popular parameter for explaining certain phenomena and for describing natural textures and it represents an important feature of textural images, hence, it is used to characterize roughness and self-similarity in a picture. This feature is used in texture description and classification, shape analysis and other computer vision and image processing problems.

III. THE BOX COUNTING METHOD

Consider a bounded set **A** in Euclidean n-space. The set **A** is said to be self-similar when **A** is the union of **N** distinct (non-overlapping) copies of itself, each of which has been scaled down by a ratio (r) in all coordinates. Simply the relation gives ***the fractal*** or ***similarity dimension*** of **A** is,

$$1 = Nr^D \quad \text{or} \quad D = \log N / \log (1/r), \ldots(4)$$

Natural ***fractal*** surfaces do not, in general, possess this deterministic self- similarity. Instead, they exhibit *statistical* self- similarity, that is, they are composed of **N** distinct subset, each of which is scaled down by a ratio **r** from the original and it is identical in all *statistical aspects* to the scaled original. ***The fractal (similarity) dimension*** for these surfaces is also given by (4). While the definition of ***fractal dimension*** by self-similarity is straightforward, it is often difficult to estimate directly from image data, However, a related measure of ***fractal dimension***, the *box dimension*, can be more easily computed from a ***fractal*** set **A** in R^n as follows.

Suppose one can cover the set **A** with n-dimensional boxes of size L_{max}. If the set **A** is scaled down by the ratio **r**, then there are ($N = R^{-D}$) subsets, and so the number of boxes of size $L = r\, L_{max}$ needed to cover the whole set is given by

$$N(L) = 1 / r^D = [Lmax / L]^D, \ldots\ldots\ldots(5)$$

The simplest way to estimate **D** from equation (5) is to divide the n-dimensional space into a grid of boxes with side length L and to count the number of non–empty boxes. ***Pickover*** and ***Khorasani*** have utilized this method to characterize speech graphs. If N (L) is computed for several values of L, then **D** can be estimated as the slope of a least squares linear fit of the data {ln (L), -ln (N (L))} [Pickover and Khorasani 86].

Voss has suggested a more elegant method to estimate ***the fractal dimension*** of an image surface **A**. Let p (m, L) be the probability that there are **m** points within a box of size L centered about an arbitrary point of **A**. for each value of L we have

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$$\sum_{m=1}^{N} P(m, L) = 1, \ldots\ldots\ldots\ldots\ldots(6)$$

Where N is the number of possible points within the box. Suppose that the total number of points in the image is M. If one overlay the image with boxes of side L, then the number of boxes with m points inside the box is (M/m) P (m, L) [Voss 86].

Therefore, the expected total number of boxes needed to cover the whole image is

$$\langle N(L) \rangle = \sum_{m=1}^{N} (M/m) P(m, L) = M \sum_{m=1}^{N} (1/m) P(m, L), \ldots\ldots\ldots\ldots(7)$$

Hence, if we let

$$N(L) = \sum_{m=1}^{L^3} (1/m) P(m, L), \ldots\ldots(8)$$

This value is also proportional to L and can be used to estimate D.

The box dimension can be calculated as follows: for every pixel in an image of size M x N. with the largest box size to be used set at L_{max}, the number of points m within each box of size L centered at the pixel (x, y, *f*(x,y)) is counted and recorded as m (L,x,y) . The centering and counting activity is restricted to pixels having all their neighbors inside the image. The image is then divided into overlapping or windows. The overlap is decided by the increment between windows. For each window, the occurrences of m (L, x, y) are accumulated over the pixels within the window and the probability distribution P (m, L) is obtained by dividing the accumulated occurrences of m (L, x, y) by the total number of pixels in the windows. For digital image surface, the values for m range from 1 to L^3 for a cube of side L. The estimate of ***the fractal dimension*** is the slope obtained by performing a least squares fit to the data set {ln (L), -ln (N (L))} with N (L) given by the above equation (8) [Keller and Chen 89].

IV. LACUNARITY & TEXTURES MEASURES

The fractal dimension of a surface images has been used as a description feature, but ***the fractal dimension*** characterizes only part of the information in the distribution P (m, L), and therefore there may exist different ***fractal sets*** have the same ***fractal dimension*** but have different "appearances" or "textures" corresponding to their different distribution P (m, L) [Keller and Chen 89]. Also, simulations of ***fractal*** surfaces have shown that even though the dimensions may remain constant, different visual textures can be achieved.

This supports the claim that ***the fractal dimension (D)*** alone does not provide sufficient information to describe natural textures. Although there is some difference in the values of ***fractal dimensions*** estimated for some textures, they are not sufficient to completely distinguish these textures. Thus ***the fractal dimension*** is not sufficient to characterize important nonfigurative texture characteristics, hence, decryption using only ***fractal dimensions*** would be useless for some natural textures, so that additional ***fractal features*** are necessary.

As an initial step toward quantifying texture, the term ***lacunarity*** is introduced to describe that characteristic of ***fractals*** of the same dimension with different appearances or texture. ***Mandelbrot*** has introduced the parameter ***lacunarity*** [Mandelbrot 82]. (***Lacuna*** in Latin means ***gap***). Although the qualitative visual effect of changing ***lacunarity*** at fixed ***fractal dimension*** value is quit striking, to date there have been no quantitative measurements of ***lacunarity***. ***Mandelbrot*** offers several alternative definitions. One derives from the width of the distribution P (m, L) at fixed L. The most useful definition for this term being

$$Lac = E\left(\left((M / E(M)) - 1\right)^2\right), \ldots\ldots\ldots(9)$$

Where M is the mass of the ***fractal set***, and E (M) is the expected mass, this definition measures the discrepancy between ***the fractal mass*** and the expected mass.

Voss suggested calculating ***lacunarity*** from the same probability distribution P (m, L) used for the estimation of ***fractal dimension*** [Voss 86].

Letting

$$M(L)=\sum_{m=1}^{N} mP(m,L), \ldots\ldots\ldots\ldots\ldots\ldots(10)$$

And

$$M_2(L)=\sum_{m=1}^{N} m^2 P(m,L), \ldots\ldots\ldots\ldots(11)$$

Lacunarity was defined as

$$Lac2(L)=\frac{M^2(L)-(N(L))^2}{(M(L))^2}, \ldots\ldots(12)$$

The probability P (m, L) contains the average information of the mass distribution of a ***fractal set***. According to [Mandelbrot 82], ***lacunarity*** is closely related to this distribution of mass. While the above definition of ***lacunarity*** works well for large texture areas, it does not provide adequate separability for smaller patches encountered in a segmentation process. A second measure of Lacunarity, based on P(m,L), which can be used for texture description and segmentation, was given by [Keller & Chen 89].

For each value of L, we have

$$Lac1(L)=\frac{M(L)-N(L)}{M(L)+N(L)}, \ldots\ldots\ldots\ldots(13)$$

Where M(L) is the average mass density within a box of side L and N(L) is proportional to the number of boxes of side L needed to cover ***the fractal set***. Actually, N(L) equals the number of boxes needed to cover the set divided by the total number of pints in the set , i.e.(L) is the fraction of a box that one point occupies.

V. FRACTAL FEATURES VERSUS THE TEXTURE ATTRIBUTES

The adopted ***fractal features*** (i.e. ***fractal dimension and lacunarity***) will be tested to be ensuring that they are sensitive to texture attributes and, hence fore, they can be used as efficient discriminators in the texture description process.

In order to perform ***the fractal features*** evaluation a sequence of simulated fractal textures was generated by using a ready-made software package called "The ***Fractal Design Painter***". The generated fractal patterns differ in three textures attributed (i.e. power, softness and feature size), the dynamic ranges of these texture attributes are:

Feature Size ranges from 0 to 100%

Power ranges from –300% to 100%, and

Softness ranges from 0 to 100%

Taking into consideration the fact that more reliable conclusion requires considering all, as much as possible, values of the above textures attributes. This led us to consider all

The cases listed below in table (1), these cases were categorized into three different test groups, and each group considers the effect of one texture attribute.

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Table (1): Test groups considered in the quantitative evaluation.

Tests	Softness%	Power %	Feature Size%
Test 1	0 to 100 step 5	-300	0
	0 to 100 step 5	-300	50
	0 to 100 step 5	-300	100
	0 to 100 step 5	-100	0
	0 to 100 step 5	-100	50
	0 to 100 step 5	-100	100
	0 to 100 step 5	100	0
	0 to 100 step 5	100	50
	0 to 100 step 5	100	100
Test 2	0	-300 to 100 step 20	0
	0	-300 to 100 step 20	50
	0	-300 to 100 step 20	100
	50	-300 to 100 step 20	0
	50	-300 to 100 step 20	50
	50	-300 to 100 step 20	100
	100	-300 to 100 step 20	0
	100	-300 to 100 step 20	50
	100	-300 to 100 step 20	100
Test 3	0	-300	0 to 100 step 5
	50	-300	0 to 100 step 5
	100	-300	0 to 100 step 5
	0	-100	0 to 100 step 5
	50	-100	0 to 100 step 5
	100	-100	0 to 100 step 5
	0	100	0 to 100 step 5
	50	100	0 to 100 step 5
	100	100	0 to 100 step 5
	0	-300	0 to 100 step 5
	50	-300	0 to 100 step 5
	100	-300	0 to 100 step 5

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The size of all tested textural images was 64x64 and their gray scale was 256. The total number of the tested images was 567. The values of fractal dimension and lacunarity corresponding to each of the above 567 textural images were estimated by using the box counting method with maximum size of the box is between 21-61

In the illustration of experimental study, we restrict the above cases to illustrate the cases listed below in table (2). This table illustrates the selected values of the softness, power and feature size for the three test groups. It is obvious that these groups contain 189 textural images.

Table (2): Selected cases for illustration scheme

Tests	Softness%	Power %	Feature Size%
Test 1	0 to 100 step 5	-300	50
	0 to 100 step 5	-100	50
	0 to 100 step 5	100	50
Test 2	0	-300 to 100 step 20	50
	50	-300 to 100 step 20	50
	100	-300 to 100 step 20	50
Test 3	50	-300	0 to 100 step 5
	50	-100	0 to 100 step 5
	50	100	0 to 100 step 5

The visual scenes of three of the following selected cases for textural images are presented in figures (1),(2) and (3).

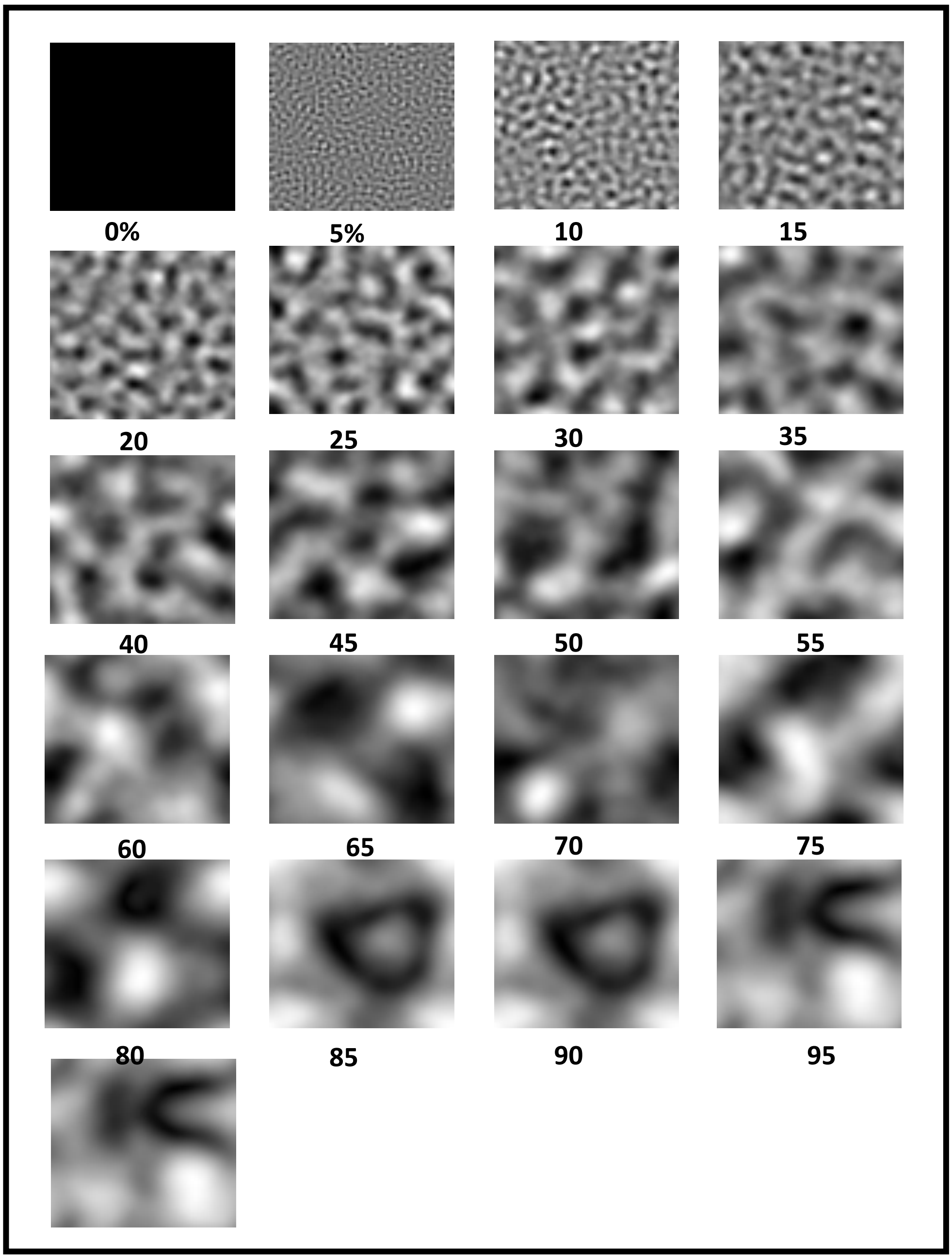
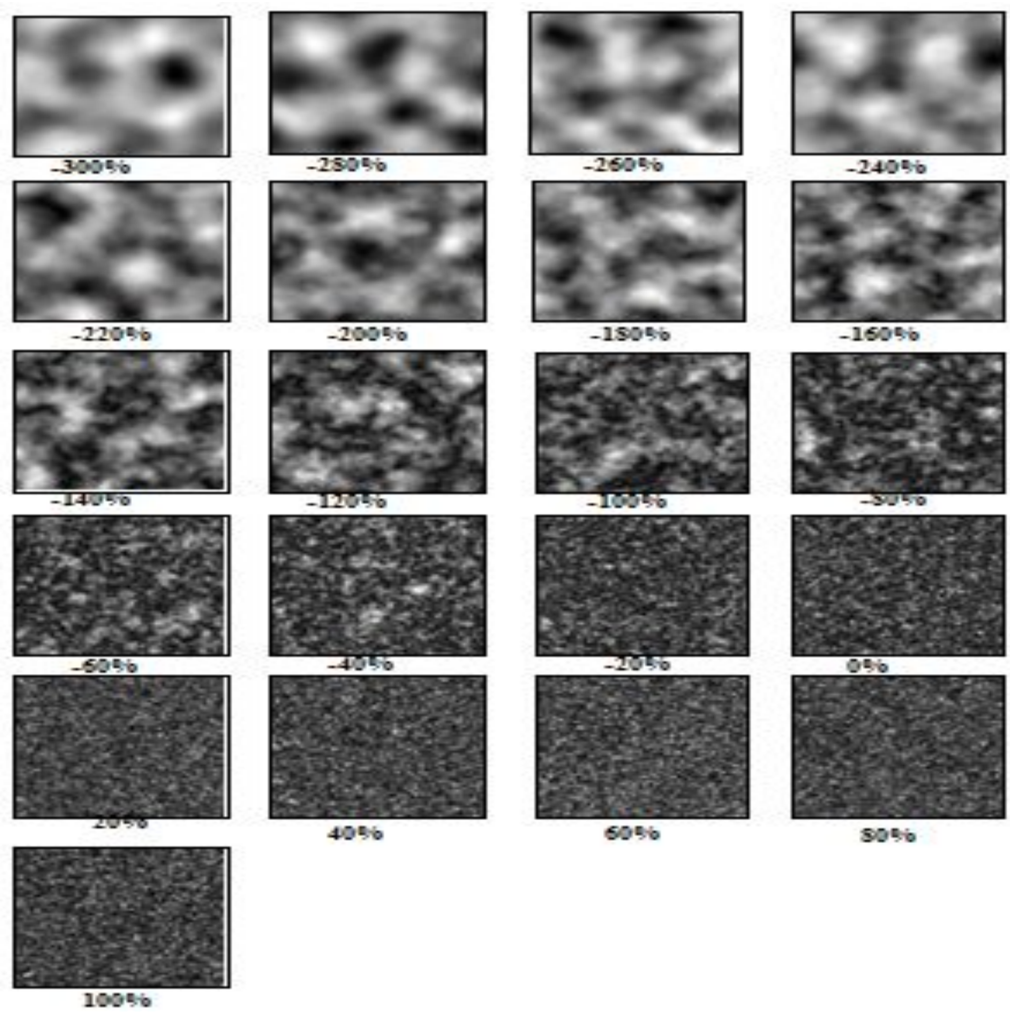


Fig. (1): The visual scene of the *fractal* patterns for the cases (power= -300% & softness=50%) and different values of feature size.

100%



<u>Fig. (2):</u> The visual scene of the *fractal* patterns for the cases (feature size=50% & softness=100%) and different values of power.

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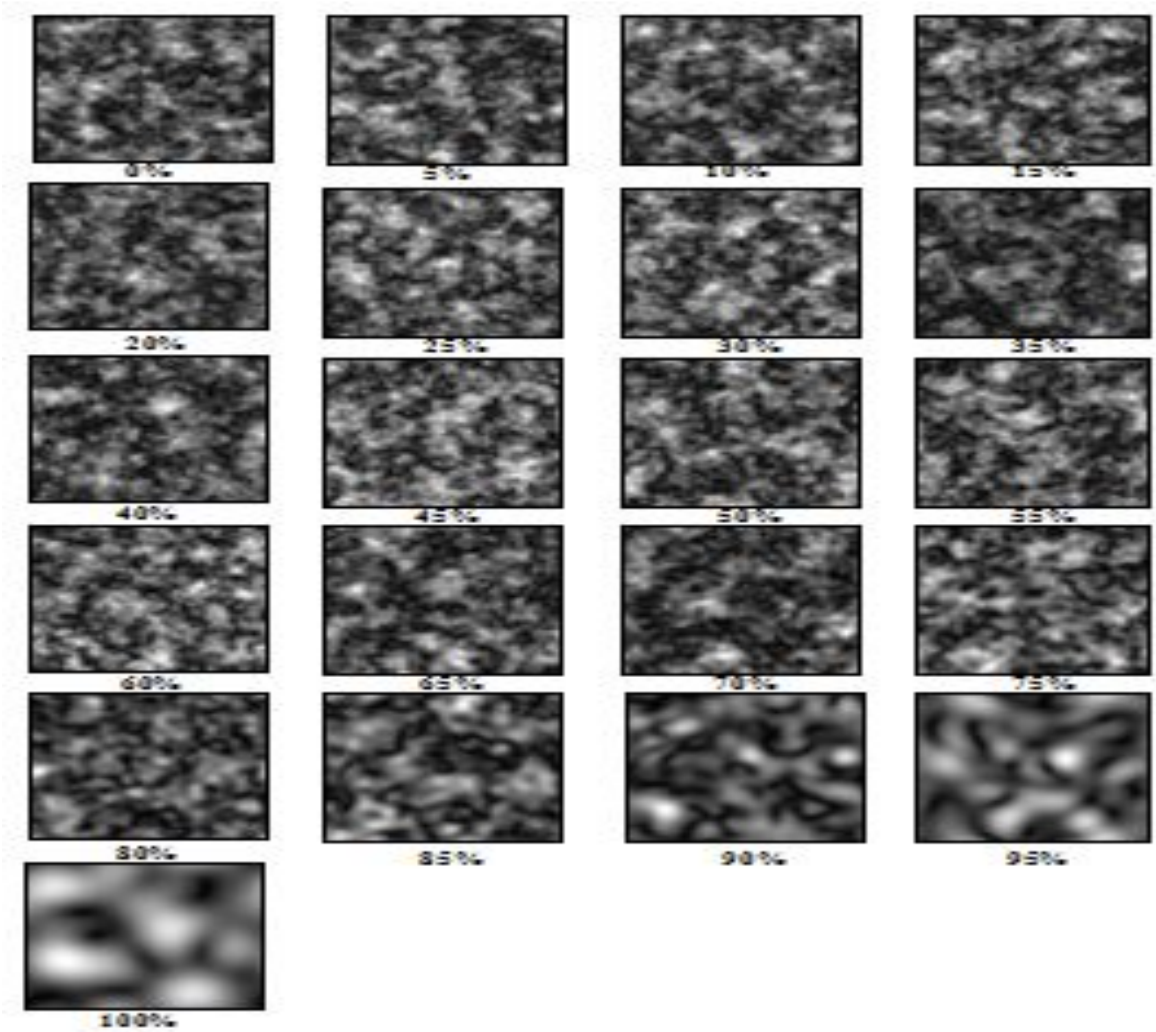
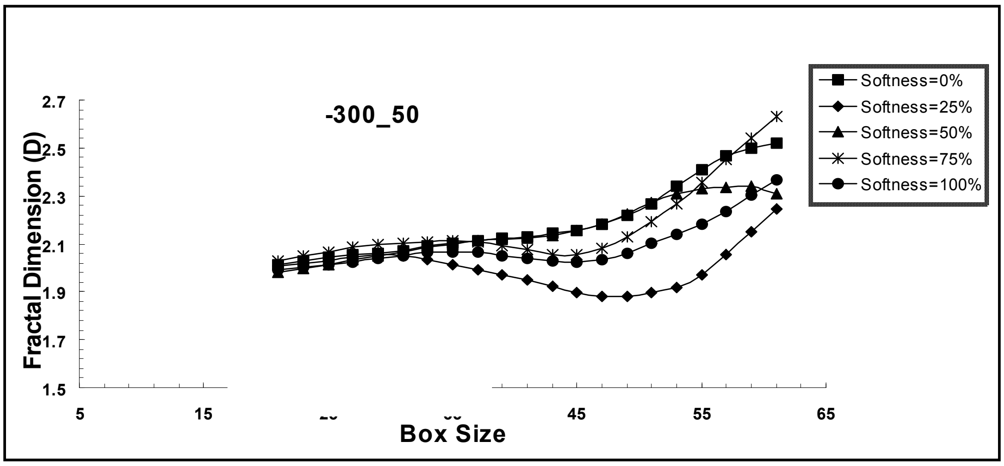
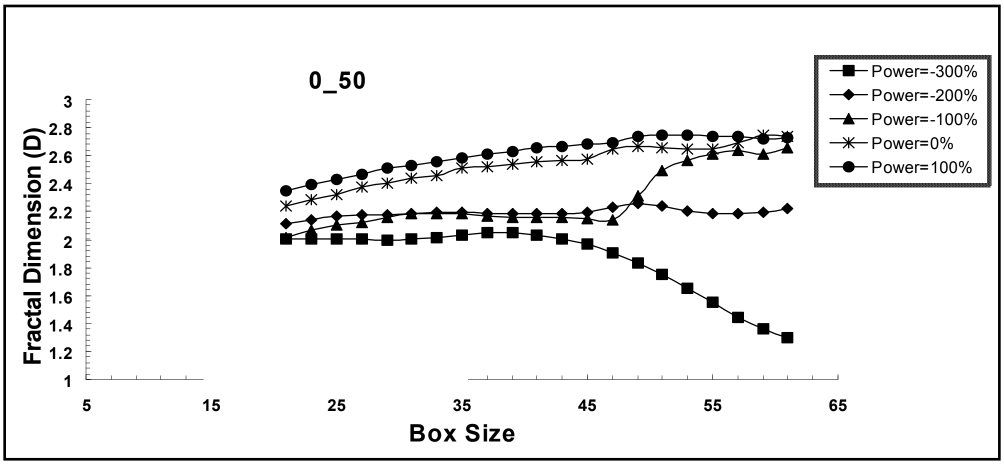


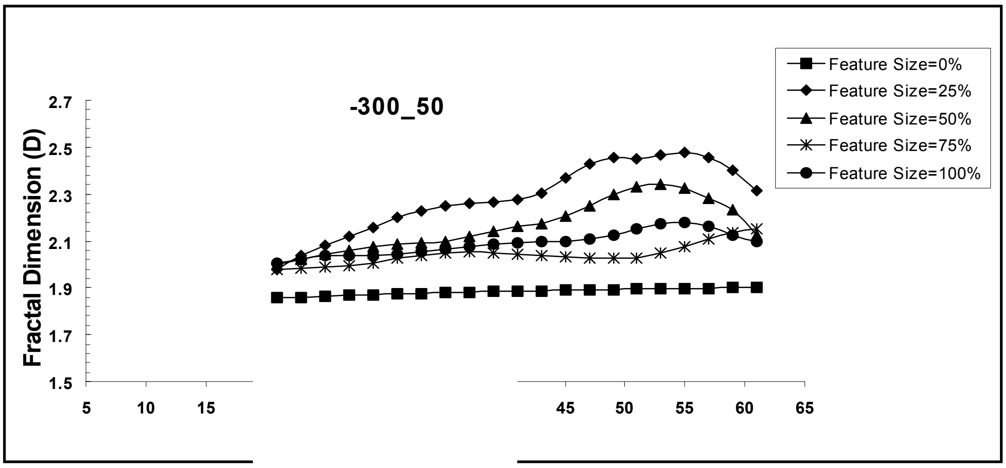
<u>Fig. (3):</u> The visual scene of the *fractal* patterns for the cases (power=-100% & feature size=50%) and different values of softness.



(a)

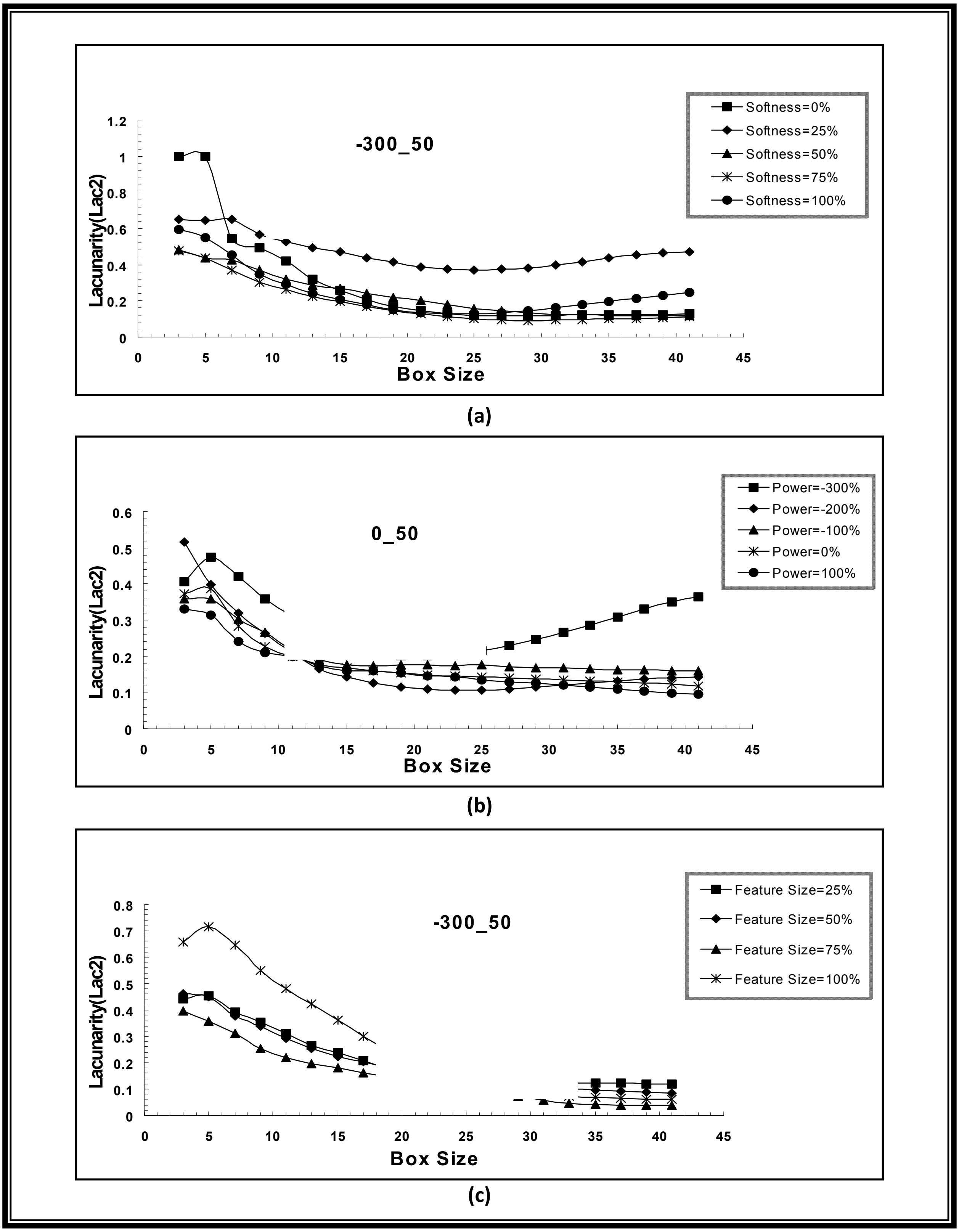


(b)



(c)

Fig. (4): The effect of the texture attributes on the aspect of *the fractal dimension (D)* for the cases (a) power = -300% and feature size =50% and different softness values. (b) feature size =0% , softness =50% and different power values. (c) power = -300% and softness =50% and different feature size values.



<u>Fig. (5):</u> **The effect of the texture attributes on the aspect of *the lacunarity (Lac2)* for the cases (a) power = -300% and feature size =50% and different softness values. (b) feature size =0% , softness =50% and different power values. (c) power = -300% and softness =50% and different feature size values.**

VII. RESULTS & DISCUSSION

It is quit obvious that the subjective appearance of the textural properties is significantly depends on the softness, feature size and power,. The calculation of ***the fractal features*** was performed by using the box counting method, and the maximum size of the box was taken variable, the results, as expressed in figures (4) and (5) have shown that the calculated ***fractal features i.e. fractal dimension and lacunarity*** are significantly differs with the variation of textural attributes.

We noticed that the Lac2 values showed instable behavior which may be due to statistical instability arises when the number of tested samples becomes poor.

There is also a noticeable translation and rotation invariant for all samples used in the robustness test in spite of the random location and angle of rotation selection.

Regarding the scale invariant there are a little differences in the estimated values for *fractal dimension and lacunarity*, that is as the size of the sub image changes (increase, the values of these fractal features are changed also and the reason for that is "***the real fractal set*** was defined to be self- similar in all scales no matter how much small or large they are, the fractal features (including *fractal dimension and lacunarity*) are strongly dependant upon the concept of self-similarity and scale–invariant, so that the results for scale-invariant are expected because of the training sub images can not be *real fractal sets*"

In spite of the little variations in the values of *fractal dimension and lacunarity* in the case of scale-invariant, we can say that these fractal features are robust.

These figures illustrate that the behaviors of ***fractal dimension and lacunarity***, as a function of L_{max} is different from pattern to another. This result led the adopting to idea that the different values of ***fractal dimension and lacunarity*** (for different box size) could be utilized to construct a ***fractal dimension and lacunarity*** feature vectors which good be exploited as an efficient discriminating ***fractal feature*** in the segmentation, classification and description processes.

VIII. CONCLUSIONS

The texture attributes (i.e. softness, power and feature size) affects subjectively, the visual scene of the textural images; hence changing the values of these texture attributes led to different aspects of ***the fractal features (fractal dimension and Lacunarity).***

The fractal feature Lacunarity Lac1 shows better sensitivity to texture attributes than Lac2.

The calculated ***fractal features (i.e. fractal dimension and Lacunarity)*** by using our modified box counting method are very sensitive to the maximum size of the box, this led to adopting the idea of using different box size to evaluate these ***fractal dimension and Lacunarity*** vectors as a feature vectors that can be easily used in the discrimination ,segmentation and classification processes.

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