Assessment and enhancement of voltage stability based on reactive power management using UPFC

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Abstract: This paper presents an algorithm to minimize the active power loss based on voltage stability using Newton Rapshon power flow analysis. Active power losses are mitigated by the management of reactive power via FACTs (UPFC). The suitable location for VAR management has been obtained via sensitivity analysis. An algorithm for power flow study with FACTS devices is proposed based on power injection model of UPFC. The developed algorithm has been implemented on the modified IEEE 30-bus system. The obtained results indicate that the active power loss is minimised and voltage stability is improved using UPFC facts devices.

Key words: Reactive power control; Voltage stability; power flow; Unified power flow compensator; Sensitivity analysis.

1. INTRODUCTION

During recent years, the planning and the operation of large interconnected power systems while improving system stability and security have become important concerns in the daily operation of modern power networks. Some of the serious issues are the power quality, transmission loadability, congestion management, power losses and voltage stability. Many high-voltage transmission systems are operating below their thermal ratings due to constraints such as voltage and stability limits. Most large power system blackouts, which occurred worldwide over the last twenty years, are caused by heavily stressed system with large amount of real and reactive power demand and low voltage condition. When the voltages at the system buses are low, the losses will also be increased. As several blackouts around the world have been related to voltage phenomena much more interest has been devoted by planning engineers to the voltage stability as well as to reduce power losses.

To overcome these issues Electric Power Research Institute (EPRI) has presented a new technology known as FACTS. Flexible AC Transmission Systems (FACTS) [1] incorporates the power electronic based equipments and other static controllers to enhance controllability. The voltage stability problem of transmission system and cheap power transfer may improve by use of the Flexible AC Transmission Systems (FACTS) controllers. It also allows increasing the usable transmission capacity to its maximum thermal limits. The rapid development of the FACTS technology [2] attracts utility to use their flexibility and ability to effectively control power system dynamics. Unified power flow controller (UPFC) is the most comprehensive multivariable flexible AC transmission system (FACTS) controller [3]. Simultaneous control of multiple power system variables with UPFC may advantageous to power system such as minimization of transmission losses, elimination of line over loads and improvement of voltage stability [4]. Jizhong et al. [5] presents new SVC model to loss minimization and voltage stability improvement and comparing it with VAR compensation with general SVC model. Tarik and Kamwa [6] presents Preventive control approach for voltage stability improvement using voltage stability constrained optimal power flow based on static line voltage stability indices.

Power flow study [7-8] or also known as load-flow study is an essential tool which involves numerical analysis applied to a power system in normal steady-state operation. A power flow study normally uses simplified notation such as single-line diagram and per-unit system, and it also takes into consideration the reactive and real powers. The term “power flow” refers to the flows of real and reactive power that occur during steady state condition in a power system. The Facts devices are incorporated in the conventional power flow algorithm using power injection based modelling of device. A comprehensive Newton-Raphson UPFC model for the quadratic power flow solution of practical power networks is presented by Fuerte et al. [9]. Control setting of unified power flow controllers through Robust Load Flow Calculation is presented by Fang and Ngan [10]. Application of UPFC for enhancement of voltage profile and minimization of losses using fast voltage stability index is presented by Kumar and Renuga [11].

In this paper, the selection of the best possible location for installation of UPFC is carried out with an objective of reducing the losses and improving the voltage stability using perturbation method of sensitivity analysis. A mathematical model for UPFC which is referred as power injection model is incorporated in Newton rapshon load flow analysis. The proposed methodology is tested on modified IEEE 30 Bus test systems in stressed conditions and compares the results with the result obtained through power flow without UPFC.

2. MODELLING OF UPFC
UPFC is a combination of static synchronous compensator (STATCOM) and a static series compensator (SSSC), which are coupled via a common DC link, to allow bi-directional flow of real power between the series output terminals of the SSSC and the shunt output terminals of the STATCOM, and are able to control the transmission line voltage, impedance, and angle, and the real and reactive power flow in the line with line compensation without an external electric energy source [12-13]. The physical model of UPFC is shown in Fig. 1.

The proposed UPFC model and all of these elements can easily be incorporated into any standard load flow and optimal power flow programs. Active and reactive power flows of the line with UPFC are:

\[
P_{tf} = G'_{tf}(V_f^2 + V_f V_c \cos \delta_{fs}) + G''_{tf}(V_f V_c \cos \delta_{fs}) + G_p'(V_f V_p \cos \delta_{fp}) - B'_{tf}(V_f V_c \sin \delta_{fs}) - B_p'(V_f V_p \sin \delta_{fp})
\]

\[
Q_{tf} = G''_{tf}(V_f V_c \sin \delta_{fs}) + G'_p(V_f V_p \sin \delta_{fp}) + B'_p(V_f V_p \cos \delta_{fp})
\]

(1)

Similarly,

\[
P_{tf} = G'_{tf}(V_f V_c \cos \delta_{tf} + V_f V_s \cos \delta_{ts}) + G''_{tf} V_s^2 - B'_{tf}(V_f V_c \sin \delta_{tf} + V_f V_s \sin \delta_{ts})
\]

\[
Q_{tf} = -G''_{tf}(V_f V_c \sin \delta_{tf} + V_f V_s \sin \delta_{ts}) + B'_t V_s^2 + B'_t(V_f V_c \cos \delta_{tf} + V_f V_s \cos \delta_{ts})
\]

(2)

The injected real power at bus-f (P_{finj}) and reactive power (Q_{finj}) of a transmission line having a UPFC are as follows,

\[
P_{finj}^{UPFC} = V_f^2 G''_{ff} + V_f V_f(G'_f \cos \delta_{fs} + B'_f \sin \delta_{fs}) + V_f V_p(G'_p \cos \delta_{fp} - B'_p \sin \delta_{fp})
\]

\[
Q_{finj}^{UPFC} = V_f^2 B''_{ff} + V_f V_f(G'_f \sin \delta_{fs} + B'_f \cos \delta_{fs}) + V_f V_p(G'_p \cos \delta_{fp} + B'_p \sin \delta_{fp})
\]

(3)

Similarly, the real power and reactive power injections at bus-t are,

\[
P_{tinj}^{UPFC} = V_f^2 G'_t + V_f V_f(G'_f \cos \delta_{tf} + B'_f \sin \delta_{tf}) + V_f V_p(G'_p \cos \delta_{ts} - B'_p \sin \delta_{ts})
\]

\[
Q_{tinj}^{UPFC} = V_f^2 B'_t + V_f V_f(G'_f \sin \delta_{tf} + B'_f \cos \delta_{tf}) + V_f V_p(G'_p \cos \delta_{ts} - B'_p \sin \delta_{ts})
\]

(4)

Where,

\[
G_{ff} = \frac{V_f^2}{1 + 2z_s V_f} + \frac{1}{z_s} G_{tt} + j B_{tt} = -\frac{V_t V_f Z_s}{1 + 2z_s V_f} G_{tt} + j B_{tt} = -\frac{V_f Z_s}{1 + 2z_s V_f} G_{tt} + j B_{tt}
\]

(5)

Where \(Y = 1/Z\) and \(Z = R + jX\) = transmission line impedance and \(\delta_t = \delta_f, \delta = -\delta_f\).
2.1 Power Flow Equations with facts devices

The load flow equations with FACTS devices, can then be obtained and referred directly as for a generic case, it is assumed that FACTS device is embedded in a transmission line between node-$f$ and node-$t$. Therefore, for the FACTS device embedded transmission line, the load flow equations can be expressed as follows:

\[ P_i = P_{gi} - P_{li} + P_{fi} \quad \text{and} \quad Q_i = Q_{gi} - Q_{li} + Q_{fi} \]  \hspace{1cm} (6)

Where \( i \) = 1, 2, ..., \( n \). And for lines without FACTS devices (if \( i \neq f \) and \( t \));

\[ P_i = P_{gi} - P_{li}, \quad Q_i = Q_{gi} - Q_{li} \]  \hspace{1cm} (7)

Where \( P_{fi} (= P_{mfi}) \) and \( Q_{fi} (= Q_{mfi}) \) are the FACTS devices real and reactive powers respectively injected at bus \( i \) (\( i = f \) & \( t \)). After FACTS devices are added in the transmission line between buses \( f \) & \( t \), injection power should be added to load flow equation. Therefore, at bus \( f \), load flow equation becomes,

\[ P_{df} - P_{df} = \sum_{j=1}^{n} V_j Y_{ij} V_j \cos(\delta_f - \delta_j - \theta_{ij}) + P_{inf} \]

\[ Q_{df} - Q_{df} = \sum_{j=1}^{n} V_j Y_{ij} V_j \sin(\delta_f - \delta_j - \theta_{ij}) + Q_{inf} \]  \hspace{1cm} (8)

Similarly, for bus \( t \),

\[ P_{dt} - P_{dt} = \sum_{j=1}^{n} V_j Y_{ij} V_j \cos(\delta_t - \delta_j - \theta_{ij}) + P_{int} \]

\[ Q_{dt} - Q_{dt} = \sum_{j=1}^{n} V_j Y_{ij} V_j \sin(\delta_t - \delta_j - \theta_{ij}) + Q_{int} \]  \hspace{1cm} (9)

Where \( n \) is the total number of buses. \( P_{inf}, Q_{inf}, P_{int} \), and \( Q_{int} \) (\( \forall i \)) are the injected real and reactive power at node-$f$ and node-$t$ and their values for, UPFC, discussed in the above section.

2.2 Operating Constraints of the UPFC

According to the operating principle of the UPFC, it can control the voltage at bus \( f \) and the active and reactive power flow for the line \( l \) (between bus-$f$ and bus-$t$ in which the UPFC is installed). The voltage constraint of the UPFC is

\[ V_f - V_f^{\text{Spec}} = 0 \]  \hspace{1cm} (10)

is the specified voltage control reference at bus \( f \) in the implementation, the above equality constraint is replaced by the following inequality constraints, Where \( V_f^{\text{Spec}} \)

\[ V_f^{\text{Spec}} - \varepsilon \leq V_f \leq V_f^{\text{Spec}} + \varepsilon \]  \hspace{1cm} (11)

Where \( \varepsilon \) is a specified very small value. The active and reactive power flow control constraints of the UPFC are:

\[ \Delta P_{ft} = P_{ft} - P_{ft}^{\text{Spec}} = 0 \]

\[ \Delta Q_{ft} = Q_{ft} - Q_{ft}^{\text{Spec}} = 0 \]  \hspace{1cm} (12)

Where, \( P_{ft}^{\text{Spec}} \) and \( Q_{ft}^{\text{Spec}} \) are specified active and reactive power flows, respectively.

3. POWER FLOW SOLUTION BY NEWTON-RAPHSON’S METHOD

Step1. Prepare the database for the system including line data, bus data, generator data and tap setting of the transformers. Line data includes the information of the lines such as resistance, reactance and shunt admittance. Bus data includes the information of the generators, loads connected at each and every bus. The generator data includes real and reactive power generation limits.

Step2. Formation of Y bus using line resistance, reactance, shunt elements and tap changing ratio

\[ [I] = [Y][V] \]  \hspace{1cm} (13)

Step3. Assume suitable values of voltage magnitude at all the buses excluding swing bus and its angle for all the buses, also set the error for calculated active and reactive power.

Step4. Calculate real and reactive power using formula for all buses using equations

\[ P_i = \sum_{i=1}^{n} |V_i||Y_{ij}||V_j| \cos(\delta_i - \delta_j - \theta_{ij}) \]

\[ Q_i = \sum_{i=1}^{n} |V_i||Y_{ij}||V_j| \sin(\delta_i - \delta_j - \theta_{ij}) \]  \hspace{1cm} (14)
Step 5. Calculate error for real and reactive power between specified and calculated for load buses and only real power for voltage control buses. If it is within tolerable limit go to Step 8, else continue the next steps.

Step 6. Calculate Jacobian matrix using equations

\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} = 
\begin{bmatrix}
J_{11} & J_{12} \\
J_{21} & J_{22}
\end{bmatrix} \begin{bmatrix}
\Delta \delta \\
\Delta |V|
\end{bmatrix}
\]  

(15)

Step 7. Calculate voltage magnitude and angle increment using the following equation (except reference bus)

\[
\begin{bmatrix}
\Delta |V| \\
\Delta \delta
\end{bmatrix} = |J|^{-1} \begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}
\]  

(16)

Step 8. Calculate new bus voltage magnitude and its angle on all buses (except reference bus) and update solution:

\[
|V_{\text{new}}| = |V_{\text{old}}| + \Delta V
\]

(17)

\[
\delta_{\text{new}} = \delta_{\text{old}} + \Delta \delta
\]

(18)

Step 9. Check whether \( \| \Delta V \| < \varepsilon \) and \( \| \Delta \delta \| < \varepsilon \). If not go to Step 4. If so, problem is solved.

### 3.1 Implementation of UPFC in Newton-Raphson Power Flow Solution

Let us consider that a UPFC is connected between bus-\( f \) and bus-\( t \), subject to control of voltage at bus-\( f \) (\( V_f^{\text{spec}} \)) and active power (\( P_f^{\text{spec}} \)) and reactive power (\( Q_f^{\text{spec}} \)) flow in the line \( f-t \) respectively. In the presence of UPFC devices, the linearized power flow equations must be combined with the linearized system of equations corresponding to the rest of the network. A compact Newton-Raphson power flow algorithm is presented as follows:

\[
F(X)^i = J^i \cdot \Delta X^i
\]

(19)

Where \( \Delta X \) is the solution vector and \( J \) is the matrix of partial derivatives of \( F(X) \) with respect to \( X \), Jacobian matrix, and they can be calculated as:

\[
F(X) = 
\begin{bmatrix}
\Delta P_f \\
\Delta P_t \\
\Delta Q_f \\
\Delta Q_t \\
\Delta P_{ft} \\
P_{\text{PE}}
\end{bmatrix},
\Delta X = 
\begin{bmatrix}
\Delta \delta_f \\
\Delta \delta_t \\
\Delta \delta_{st} \\
\Delta \delta_{sp} \\
\Delta \delta_{f}, \Delta \delta_{t}
\end{bmatrix}
\]  

(20)

And

\[
J = 
\begin{bmatrix}
\frac{\partial P_f}{\partial \delta_f} & \frac{\partial P_f}{\partial V_f} & \frac{\partial P_f}{\partial Q_f} & \frac{\partial P_f}{\partial P_t} & \frac{\partial P_f}{\partial Q_t} & \frac{\partial P_f}{\partial \delta_{st}} & \frac{\partial P_f}{\partial \delta_{sp}} & \frac{\partial P_f}{\partial \delta_{f}} & \frac{\partial P_f}{\partial \delta_{t}}

\frac{\partial P_t}{\partial \delta_f} & \frac{\partial P_t}{\partial V_f} & \frac{\partial P_t}{\partial Q_f} & \frac{\partial P_t}{\partial P_f} & \frac{\partial P_t}{\partial Q_f} & \frac{\partial P_t}{\partial \delta_{st}} & \frac{\partial P_t}{\partial \delta_{sp}} & \frac{\partial P_t}{\partial \delta_{f}} & \frac{\partial P_t}{\partial \delta_{t}}

\frac{\partial Q_f}{\partial \delta_f} & \frac{\partial Q_f}{\partial V_f} & \frac{\partial Q_f}{\partial Q_f} & \frac{\partial Q_f}{\partial P_f} & \frac{\partial Q_f}{\partial Q_t} & \frac{\partial Q_f}{\partial \delta_{st}} & \frac{\partial Q_f}{\partial \delta_{sp}} & \frac{\partial Q_f}{\partial \delta_{f}} & \frac{\partial Q_f}{\partial \delta_{t}}

\frac{\partial Q_t}{\partial \delta_f} & \frac{\partial Q_t}{\partial V_f} & \frac{\partial Q_t}{\partial Q_f} & \frac{\partial Q_t}{\partial P_f} & \frac{\partial Q_t}{\partial Q_t} & \frac{\partial Q_t}{\partial \delta_{st}} & \frac{\partial Q_t}{\partial \delta_{sp}} & \frac{\partial Q_t}{\partial \delta_{f}} & \frac{\partial Q_t}{\partial \delta_{t}}

\frac{\partial P_{ft}}{\partial \delta_f} & \frac{\partial P_{ft}}{\partial V_f} & \frac{\partial P_{ft}}{\partial Q_f} & \frac{\partial P_{ft}}{\partial P_f} & \frac{\partial P_{ft}}{\partial Q_t} & \frac{\partial P_{ft}}{\partial \delta_{st}} & \frac{\partial P_{ft}}{\partial \delta_{sp}} & \frac{\partial P_{ft}}{\partial \delta_{f}} & \frac{\partial P_{ft}}{\partial \delta_{t}}

\frac{\partial P_{\text{PE}}}{\partial \delta_f} & \frac{\partial P_{\text{PE}}}{\partial V_f} & \frac{\partial P_{\text{PE}}}{\partial Q_f} & \frac{\partial P_{\text{PE}}}{\partial P_f} & \frac{\partial P_{\text{PE}}}{\partial Q_t} & \frac{\partial P_{\text{PE}}}{\partial \delta_{st}} & \frac{\partial P_{\text{PE}}}{\partial \delta_{sp}} & \frac{\partial P_{\text{PE}}}{\partial \delta_{f}} & \frac{\partial P_{\text{PE}}}{\partial \delta_{t}}

\frac{\partial P_{\text{PE}}}{\partial \delta_f} & \frac{\partial P_{\text{PE}}}{\partial V_f} & \frac{\partial P_{\text{PE}}}{\partial Q_f} & \frac{\partial P_{\text{PE}}}{\partial P_f} & \frac{\partial P_{\text{PE}}}{\partial Q_t} & \frac{\partial P_{\text{PE}}}{\partial \delta_{st}} & \frac{\partial P_{\text{PE}}}{\partial \delta_{sp}} & \frac{\partial P_{\text{PE}}}{\partial \delta_{f}} & \frac{\partial P_{\text{PE}}}{\partial \delta_{t}}
\end{bmatrix}
\]  

(21)

Where, \( \Delta P_f, \Delta Q_f, \Delta P_t, \Delta Q_t \) are the active and reactive power mismatches at the terminal buses \( f \) and \( t \) respectively. \( P_f, Q_f, P_t, Q_t \) are the sum of active and reactive power flows leaving the terminal buses \( f \) and \( t \) respectively. \( \Delta P_{ft}, \Delta Q_{ft} \) are the active and reactive power flow mismatches for the line \( f-t \) respectively. And \( P_{\text{PE}} \) is the active power exchange between the converters via the common DC link.
4. VOLTAGE STABILITY ASSESSMENT

Voltage stability refer to the ability of power system to maintain steady voltages at all buses in the system after being subjected to a disturbance from a given initial operating point. The system state enters the voltage instability region when a disturbance or an increase in load demand or alteration in system state results in an uncontrollable and continuous drop in system voltage. With proper use, power flow can be an accurate tool for assessing voltage instability despite its many modelling, algorithmic, and control shortcomings. Two types of voltage instability exist in a power flow model:

1. A "loss of voltage control" voltage instability that is caused by exhaustion of reactive supply with resultant loss of voltage control on a particular set of generators, synchronous condensers, or SVC's. The loss of voltage control not only cuts off the reactive supply to a subregion requiring reactive power, but increases reactive network losses that prevent sufficient reactive supply from reaching that subregion needing reactive power. This problem may be associated with limit-induced bifurcations of a nonlinear model of the power System.

2. A "clogging voltage instability" ("radial" voltage instability) that occurs due to \( P \times X \) series reactive losses, tap-changers reaching tap limits, switchable shunt capacitors reaching susceptance limits, and shunt capacitive withdrawal due to decreasing voltage. These network reactive losses that result from the above possibilities can completely choke off the reactive flow to a subregion needing reactive supply without any exhaustion of reactive reserves and loss of voltage control on generators, synchronous condensers. This problem may be associated with a saddle-node bifurcation of a nonlinear model of the power system.

Clogging voltage instability is a well understood type of voltage instability and occurs in the distribution network, subtransmission network, and occasionally in the transmission network. It occurs due to increased transfer, and can be assessed using a P-V curve or loadability assessment methods, as discussed in Loss of voltage control instability occurs in the transmission and subtransmission system due to equipment outages as well as operating changes such as Load and generation pattern increase or Wheeling and transfer pattern increases. A system is said to be in voltage stable state if at a given operating condition, for every bus in the system, the bus voltage magnitude increases as the reactive power injection at the same bus is increased. A system is voltage unstable if for at least one bus in the system, the bus voltage magnitude decreases as the reactive power injection at the same bus is increased. It implies that if, \( V-Q \) sensitivity is positive for every bus the system is voltage stable and if \( V-Q \) sensitivity is negative for at least one bus, the system is voltage unstable.

4.1 V-Q sensitivity analysis

The \( V-Q \) sensitivity analysis mainly depends on the power-flow Jacobian matrix of equation. Kundur [14] proposed this method in 1992. It can predict voltage instability in complex power system networks. It involves mainly the computing of the smallest Eigen values and associated Eigen vectors of the reduced Jacobian matrix obtained from the load flow solution. The Eigen values are associated with a mode of voltage and reactive power variation which can provide a relative measure of proximity to voltage instability. From equation (15) the linearized steady state system power voltage equations are given by,

\[
\begin{bmatrix}
\Delta P \\
\Delta Q \\
\Delta \delta \\
\Delta V
\end{bmatrix} =
\begin{bmatrix}
J_{11} & J_{12} \\
J_{21} & J_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta [V]
\end{bmatrix}
\]

(22)

Where,

\( \Delta P \) = Incremental change in bus real power  \\
\( \Delta Q \) = Incremental change in bus reactive power  \\
\( \Delta \delta \) = Incremental change in bus voltage angle  \\
\( \Delta V \) = Incremental change in bus voltage  \\

From equation (22), let \( \Delta P = 0 \), then

\( \Delta P = 0 = J_{11} \Delta \delta + J_{12} \Delta V \)

Or

\( \Delta \delta = -J_{11}^{-1}J_{12} \Delta V \)

(23)

And

\( \Delta Q = J_{21} \Delta \delta + J_{22} \Delta V \)

(24)

Substituting Equation (23) in Equation (24)

\( \Delta Q = [J_{22} - J_{21}J_{11}^{-1}J_{12}] \Delta V \)
Or,
\[ \Delta Q = J_R \Delta V \]  
(25)

Where,
\[ J_R = [J_{22} - J_{21}J_{11}^{-1}J_{12}] \]

\( J_R \) is called the reduced Jacobian matrix of the system. Equation (25) is written as
\[ \Delta V = J_R^{-1} \Delta Q \]  
(26)

The matrix \( J_R^{-1} \) is the reduced \( V-Q \) Jacobian. Its \( i^{th} \) diagonal element is \( V-Q \) sensitivity at bus \( i \). For computational efficiency, this matrix is not explicitly formed. The \( V-Q \) sensitivities are calculated by solving equation (26). The \( V-Q \) sensitivity at a bus represents the slope of the \( Q-V \) curve at the given operating point. A positive \( V-Q \) sensitivity is indicative of stable operation; the smaller the sensitivity, the more stable the system. As stability decreases, the magnitude of the sensitivity increases, becoming infinite at the stability limit. Conversely negative \( V-Q \) sensitivity is indicative of unstable operation. A small negative sensitivity represents a very unstable operation.

The matrix \( J_R \) represents the linearized relationship between the incremental changes in bus voltage (\( \Delta V \)) and bus reactive power injection (\( \Delta Q \)). It’s well known that, the system voltage is affected by both real and reactive power variations. In order to focus the study of the reactive demand and supply problem of the system. The eigenvalues and eigenvectors of the reduced order Jacobian matrix \( J_R \) are used for the voltage stability characteristics analysis. Voltage instability can be detected by identifying modes of the eigenvalues matrix \( J_R \). The magnitude of the eigenvalues provides a relative measure of proximity to instability. The eigenvectors on the other hand present information related to the mechanism of loss of voltage stability. Modal analysis of \( J_R \) results in the following.

\[ J_R = \lambda \Phi \xi \]  
(27)

Where,
- \( \Phi \) = right eigenvector matrix of \( J_R \)
- \( \xi \) = left eigenvector matrix of \( J_R \)
- \( \lambda \) = diagonal eigenvalue matrix of \( J_R \)

Or
\[ J_R^{-1} = \Phi \xi^{-1} \]  
(28)

From equation (26)
\[ \Delta V = \Phi \xi^{-1} \Delta Q \]  
(29)

Or,
\[ \Delta V = \sum_{i=1}^{n} \frac{\xi_i \Phi_i}{\lambda_i} \Delta Q ; i=1...n. \]  
(30)

In general it can be said that, a system is voltage stable if the eigenvalues of \( J_R \) are all positive. This is different from dynamic systems where eigenvalues with negative real parts are stable. The relationship between system voltage stability and eigenvalues of the \( J_R \) matrix is best understood by relating the eigenvalues with the \( V-Q \) sensitivities of each bus (which must be positive for stability). \( J_R \) can be taken as a symmetric matrix and therefore the eigenvalues of \( J_R \) are close to being purely real. If all the eigenvalues are positive, \( J_R \) is positive definite and the \( V-Q \) sensitivities are also positive, indicating that the system is voltage stable. The system is considered voltage unstable if at least one of the eigenvalues is negative. A zero eigenvalue of \( J_R \) means that the system is on the verge of voltage instability. Furthermore, small eigenvalue of \( J_R \) determine the proximity of the system to being voltage unstable. There is no need to evaluate all the eigenvalues of \( J_R \) of a large power system because it is known that once the minimum eigenvalues becomes zeros the system Jacobian matrix becomes singular and voltage instability occurs. So the eigenvalues of importance are the critical eigenvalues of the reduced Jacobian matrix \( J_R \). Thus, the smallest eigenvalues of \( J_R \) are taken to be the least stable modes of the system. The rest of the eigenvalues are neglected because they are considered to be strong enough modes. Once the minimum eigenvalues and the corresponding left and right eigenvectors have been calculated the participation factor can be used to identify the weakest node or bus in the system.

4.2 Sensitivity Analysis for Placement of UPFC
The perturbation method to compute the sensitivity of the bus voltage. The magnitude of the bus voltage sensitivity can be expressed by the total incremental bus voltage \( \sum \Delta V_i \) which is obtained by increasing a small reactive power demand at a given load bus. The total incremental bus voltage may only include the voltage changes on several monitored buses. The bigger the value of \( \sum \Delta V_i \) the more sensitive will be the voltage at a given bus to a change of reactive demand. This means that a load bus with the large value of \( \sum \Delta V_i \) is a good candidate to be selected as a VAR compensation bus. If the maximal number of VAR compensation sites is \( m \), we can obtain \( m \) VAR compensation sites according to the values of \( \sum \Delta V_i \). Thus the corresponding sensitivity index can be expressed as

\[
S_{k}^{VQ} = \frac{\sum_{i \in NM} \Delta V_i}{\Delta Q_k}, k = 1, \ldots, ND
\]

Where:
- \( NM \): The set of the monitored buses
- \( ND \): The total number of load buses

To determine the placement of the UPFC devices, the sensitivity are computed for each load Bus in the modified IEEE 30-bus system as shown in Fig.2. The placement of UPFC is done based on this analysis i.e. here bus 24 is most sensitive and next is bus 10. Hence UPFC is placed near these locations.

Fig.2. Bar graph showing sensitivity of load buses.

5. Result and discussion

Firstly power flow is performed on a stressed IEEE 30 bus system with 135% of loading and then UPFC device has been installed on branch number 26 (bus 10-bus 17) and 33 (bus 24-bus 25). These devices are installed nearer to buses 10 and 24 respectively. The proposed algorithm has been implemented in MATLAB R2010a on a PC (Intel(R) Core TM i5 processor @1.7 GHz). The parameters of UPFC device used in this work are shown in Table 1.

<table>
<thead>
<tr>
<th>( X_s )</th>
<th>( X_p )</th>
<th>( V_s^{\text{max}} )</th>
<th>( V_p^{\text{max}} )</th>
<th>( S_s^{\text{max}} )</th>
<th>( S_p^{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>0.02</td>
<td>0.5</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Where \( X_s \), \( V_s^{\text{max}} \) and \( S_s^{\text{max}} \) are impedance, maximum voltage and maximum apparent power of series injected source respectively. \( X_p \), \( V_p^{\text{max}} \) and \( S_p^{\text{max}} \) are impedance, maximum voltage and maximum apparent power of shunt injected source respectively. Table 2 presents the solutions including voltage stability and losses of the IEEE 30-bus system with and without UPFCs. Total load: 384.860 + j174.360 at 135% increased loading.

<table>
<thead>
<tr>
<th>( T_{6,9} )</th>
<th>( T_{6,10} )</th>
<th>( T_{4,12} )</th>
<th>( T_{28,27} )</th>
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<tbody>
<tr>
<td>0.978</td>
<td>0.969</td>
<td>0.932</td>
<td>0.968</td>
</tr>
</tbody>
</table>
Table 3. Load flow solution with and without UPFC

<table>
<thead>
<tr>
<th></th>
<th>Load flow with UPFC</th>
<th>Load flow in stressed cond.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{G1}$ (MW)</td>
<td>227.288</td>
<td>228.839</td>
</tr>
<tr>
<td>$P_{G2}$ (MW)</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>$P_{G5}$ (MW)</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>$P_{G8}$ (MW)</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$P_{G11}$ (MW)</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$P_{G13}$ (MW)</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Total $P_G$ (MW)</td>
<td>402.288</td>
<td>403.839</td>
</tr>
<tr>
<td>$P_{Loss}$ (MW)</td>
<td>17.568</td>
<td>18.979</td>
</tr>
</tbody>
</table>

Fig. 3. Comparison of voltage sensitivity and losses of the two cases.

Total reduction in loss: 1.411 MW, and voltage stability is improved considerably after UPFC installation. The comparison of bus voltages and angles are given in Table 4.

Table 4. Comparison of Bus Voltages and Angles.

<table>
<thead>
<tr>
<th>Bus</th>
<th>Load flow with UPFC</th>
<th>Load flow in stressed condition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V$ (pu)</td>
<td>Phase Angle(degree)</td>
</tr>
<tr>
<td>1</td>
<td>1.060</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>1.043</td>
<td>-4.455</td>
</tr>
<tr>
<td>3</td>
<td>1.015</td>
<td>-7.829</td>
</tr>
<tr>
<td>4</td>
<td>1.005</td>
<td>-9.370</td>
</tr>
<tr>
<td>5</td>
<td>1.010</td>
<td>-11.687</td>
</tr>
<tr>
<td>6</td>
<td>1.005</td>
<td>-10.895</td>
</tr>
<tr>
<td>7</td>
<td>0.991</td>
<td>-12.174</td>
</tr>
<tr>
<td>8</td>
<td>1.010</td>
<td>-11.354</td>
</tr>
<tr>
<td>9</td>
<td>1.038</td>
<td>-14.583</td>
</tr>
<tr>
<td>10</td>
<td>1.023</td>
<td>-17.729</td>
</tr>
<tr>
<td>11</td>
<td>1.082</td>
<td>-12.461</td>
</tr>
<tr>
<td>12</td>
<td>1.044</td>
<td>-16.259</td>
</tr>
<tr>
<td>13</td>
<td>1.071</td>
<td>-14.466</td>
</tr>
<tr>
<td>14</td>
<td>1.022</td>
<td>-17.847</td>
</tr>
<tr>
<td>15</td>
<td>1.013</td>
<td>-18.206</td>
</tr>
<tr>
<td>16</td>
<td>1.023</td>
<td>-17.333</td>
</tr>
<tr>
<td>17</td>
<td>1.014</td>
<td>-17.994</td>
</tr>
<tr>
<td>18</td>
<td>0.995</td>
<td>-19.292</td>
</tr>
<tr>
<td>19</td>
<td>0.989</td>
<td>-19.638</td>
</tr>
<tr>
<td>20</td>
<td>0.996</td>
<td>-19.264</td>
</tr>
<tr>
<td>21</td>
<td>1.005</td>
<td>-18.686</td>
</tr>
<tr>
<td>22</td>
<td>1.008</td>
<td>-18.703</td>
</tr>
<tr>
<td>23</td>
<td>1.005</td>
<td>-19.318</td>
</tr>
<tr>
<td>24</td>
<td>1.008</td>
<td>-20.105</td>
</tr>
<tr>
<td>25</td>
<td>0.998</td>
<td>-19.264</td>
</tr>
<tr>
<td>26</td>
<td>0.971</td>
<td>-19.924</td>
</tr>
<tr>
<td>27</td>
<td>1.005</td>
<td>-18.340</td>
</tr>
</tbody>
</table>
### Table 5. Control Parameters of UPFC

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Line no.26(bus10-bus 17)</th>
<th>Line no.33(bus 24- bus 25)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_p$(degree)</td>
<td>-4.71</td>
<td>-5.67</td>
</tr>
<tr>
<td>$\delta_s$(degree)</td>
<td>-36.38</td>
<td>-43.25</td>
</tr>
<tr>
<td>$V_s$(pu)</td>
<td>0.053</td>
<td>0.113</td>
</tr>
</tbody>
</table>

### 7. CONCLUSION

The proposed algorithm for minimizing the transmission losses and improvement of voltage stability has been implemented. The effect of the UPFC placement shows the considerable reduction in active power loss and improves the voltage stability of the system. The proposed algorithm is simple in implementation and useful to power flow study with facts. This research work will be a very useful contribution in the field of power flow with FACTS devices in modern power systems and power industry.

### References


