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A NUMERICAL APPROACH FOR THE SOLUTION OF THE ONE PARAMETER FUZZIFIED FALKNER-SKAN EQUATION OF FLUID MECHANICS

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ABSTRACT

A numerical approach has been done to study about the velocity distribution in Falkner-Skan equation of fluid mechanics. The flow governing partial differential equations with the boundary conditions are transformed into ordinary differential equations by using suitable similarity transformation and then fuzzified using Zadeh's extension principle. The fuzzified forms of the governing equations along with the fuzzified boundary conditions are solved by fourth order Runga-Kutta shooting method. Using triangular fuzzy number, the left, right and mid values of the velocity profile are presented for various values of the involved included wedge angle in tabular form.

Keywords*: Falkner-Skan equation, Zadeh's extension principle, triangular fuzzy number, Numerical approach.*

I. INTRODUCTION

The problem of potential flow for which similar solutions exist is of great interest in fluid mechanics and wasfirst discussed in detail by S. Goldstein [1] and later by W. Mangler [2].The partial differential equations governing the flow of viscous fluids past a wedge have been reduced to an ordinary differential equation by using similarity transformations. The equation known as the Falkner-Skan [3] equation is a quasi linear third order ordinary differential equation containing one parameter β arising out of the included wedge angle. The problem is of interest due to the fact that when $\beta = 0$, ½ and 1 it gives rise to three famous problems in fluid mechanics which were solved by three different authors [4, 5, 6, 7], [8, 9] and [10] respectively.

Apart from physical considerations the Falkner-Skan equation together with the associated boundary conditions represents a problem of mathematical interest in as much as a numerical solution valid for all possible values of the paraeterβ, arising out of the included wedge angle of the boundary value problem can be sought by methods of numerical analysis. The numerical solutions would be of interest partly due to the non-linearity of the differential equation and partly due to the fact that in the earlier investigations the analysis has been restricted to particular values of the parameter.

Hazarika G. C.[11]Developed a general algorithm for solving third order two-point boundary value problem using shooting method [12, 13] and to apply the algorithm to obtain a solution of the Falkner-Skan equation valid for all possible values of the parameter β. Two other algorithms are developed, one to obtain a solution of the equation using finite-difference method [14, 15] and the other based on the Shooting method to estimate the parameter β for prescribed initial conditions.

II. THE FALKNER AND SKAN EQUATION

The boundary layer equations for plane incompressible flow for the problem described above are :

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{\partial U}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \quad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

with boundary conditions

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$$\left.\begin{aligned} y = 0 &: u = 0, v = 0 \\ y = \infty &: u = U \end{aligned}\right\} \quad (3)$$

wherex, y are Cartesian co-ordinates in plane flow, u, v are the velocity components, U is the velocity outside the boundary layer and ν is the constant coefficient of viscosity.
Introducing dimensionless variables

$$\eta = y\sqrt{\frac{m+1}{2}\frac{U}{\nu x}}$$
$$u = Uf'(\eta)$$

and

$$v = -\sqrt{\frac{m+1}{2}\frac{U\nu}{x}}\left\{f + \frac{m-1}{m+1}\eta f'\right\}$$

In the equation (2.1) we get
$$f''' - ff'' - \beta(1 - f'^2) = 0 \quad (4)$$
where $\beta = \frac{2m}{m+1}$.
The boundary conditions become

$$\left.\begin{aligned} \eta = 0 &: f = 0, f' = 0 \\ \eta = \infty &: f' = 1 \end{aligned}\right\} \quad (5)$$

where a prime denotes differentiation with respect to η and β is the included wedge angle. The equation (1) together with the boundary conditions (3) was first deduced by V. M. Falkner and S. W. Skan [3] known as the Falkner-Skan equation which goes after their names and represents the differential equation governing the flow past a wedge.

III. BASIC CONCEPTS OF FUZZY SET THEORY

Fuzzy Sets and Membership Function:
Let X be a universal set. A fuzzy set $\tilde{A}$ in X is characterized by its membership function denoted by $\mu_{\tilde{A}}$, i.e.,

$$\mu_{\tilde{A}}: X \to [0,1]$$

and$\mu_{\tilde{A}}$ is known as the membership grade of element x in fuzzy set $\tilde{A}$ for each $x \in X$.

If $X = \{x\}$ is a collection of objects denoted generically by x, then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$$

where each pair $(x, \mu_{\tilde{A}}(x))$ is called a singleton.
Fuzzy Number [16]:
A fuzzy number is a convex normalized fuzzy set defined on R whose membership function is piecewise continuous.
A triangular fuzzy number A can be defined as atriplet $[a, b, c]$. Its membership function is defined as:

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, a \le x \le b \\ \frac{c-x}{c-b}, b \le x \le c \end{cases}$$

A trapezoidal fuzzy number A can be expressed as $[a, b, c, d]$and its membership function is defined as:

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, a \le x \le b \\ 1, \quad b \le x \le c \\ \frac{d-x}{d-c}, c \le x \le d \end{cases}$$

Zadeh's Extension Principle [17]:

A crisp function $f: X \to Y$ is said to be fuzzified when it is extended to act on fuzzy set defined on X and Y. i.e.,

$$\tilde{f}: \tilde{X} \to \tilde{Y}$$

and its inverse has the form $\tilde{f}^{-1}: \tilde{Y} \to \tilde{X}$.

The extension principle states that for a given crisp function $f: X \to Y$ there induces two functions $\tilde{f}$ and $\tilde{f}^{-1}$ which are defined above for which membership functions are given by

$$\mu_{[\tilde{f}(A)](y)} = \sup_{x:y=f(x)} \mu_A(x), \text{ for all } A \in \tilde{X}$$

and

$$\mu_{[\tilde{f}^{-1}(B)](x)} = \mu_B(\tilde{f}(x)), \text{ for all } B \in \tilde{Y}.$$

IV. FUZZIFICATION OF THE ABOVE EQUATION

Every physical problem is inherently biased by uncertainty. There is often a need to model, solve and interpret the problems one encounters in the world of uncertainty. In general, science and engineering systems are modelled to ordinary and partial differential equations, but the type of differential equation depends upon the application, domain, complicated environment, the effect of coupling and so on. In recent years, this subject has become an important area of research due to its wide range of applications in various disciplines, namely physics, chemistry, applied mathematics, biology, economics, and in engineering systems such as fluid mechanics, viscoelasticity, civil, mechanical, aerospace, chemical etc. [18].

In general, parameters, variables and initial conditions involved in the model are considered as crisp or defined exactly for easy computation. However, rather than the particular value, we may have only the vague, imprecise and incomplete information about the variables and parameters being a result of errors in measurement, observations, experiment, applying different operating conditions, or it may be maintenance-induced errors, which are uncertain in nature. So, to overcome these uncertainties and vagueness, one may use interval and fuzzy set theory. Interval and fuzzy set theory refers to the uncertainty when we may have lack of knowledge or incomplete information about the variables and parameters.

Several authors have used the concept of fuzzy set theory in the field of differential equations.Fuzzy differential equations are used in modelling problems in science and engineering. It represents a proper way to model dynamical systems under uncertainty and vagueness.Kaleva [19] first gave the idea of fuzzy differential equations. His paper dealt with fuzzy set-valued mappings of a real variable whose values are normal, convex, upper semi-continuous and compactly supported fuzzy sets in $\square^n$. Gomes et al. [20] discussed fuzzy differential equations in various approaches. Chakravertyet al. [18] and Kermani and Saburi [21] have discussed some numerical methods for fuzzy fractional differential equation and fuzzy partial differential equation respectively. Chalco-Cano et al.[22] also studied about the solution of fuzzy differential equations.

There is a need to extend concepts from the classical set theory to fuzzy set theory. The fuzzification process can be done by using various methods. Extension-principle and Max-Min principle of Zadeh are the main tools for fuzzification of crisp relationships [23]. Mizukoshiet al. [24] studied the fuzzy differential equations and the extension principle. They studied the Cauchy problem for differential equations, considering the parameters and the initial conditions given by fuzzy sets. They approached the fuzzy differential equations in two different ways- using a family of differential inclusions and the Zadeh's extension principle for the solution of the model. The solutions of the said problem obtained by both the methods were found same. Ahmed et al. [25] also studied a fuzzy fractional differential equation and present its solution using Zadeh's extension principle.

We have used Zadeh's extension principle to extend the above ordinary differential equation into fuzzy form as below:

$$\tilde{f}''' - \tilde{f}'\tilde{f}'' - \tilde{\beta}\left(1 - \tilde{f}'^2\right) = 0 \qquad (6)$$

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Corresponding boundary conditions are-

$$\left.\begin{array}{l}\eta = 0: \tilde{f} = 0, \tilde{f}' = 0 \\ \eta = \infty: \tilde{f}' = 1\end{array}\right\} \quad (7)$$

V. RESULTS AND DISCUSSION

The two guessed values needed to estimate $\tilde{f}''(0)$are chosen by trial and error methods. Once the correct values are obtained for some cases e.g. for some prescribed β in estimation of $\tilde{f}''(0)$then it becomes easier to choose the guessed values for any other values of $\tilde{f}''(0)$.

Table 1 to Table 3 give the solution for$\tilde{\beta} = [-0.1, -0.1, -.1], [0, 0, 0], [0.25, 0.25, 0.25], [0.50, 0.50, 0.50], [1.00, 1.00, 1.00]$ and $[1.25, 1.25, 1.25]$obtained by the Shooting method are in good conformity with those of crisp values.

The system of fuzzified ordinary differential equation (6) together with the boundary conditions (7) is solved numerically by fourth order Runge-Kutta shooting method. We have considered the triangular fuzzy number in solving the equations. The values of the parameter$\tilde{\beta}$ involved in the problem are taken as given above.

The following tables give the values of velocity for the values $\tilde{\beta}$ given against values of η.
Fuzzified velocity for different values of $\tilde{\beta}$

Table 1

	$\tilde{\beta}$=[-0.10,-.10,-0.10]			$\tilde{\beta}$=[0, 0, 0]		
η	$\tilde{f}'$(left)	$\tilde{f}'$(mid)	$\tilde{f}'$ (right)	$\tilde{f}'$(left)	$\tilde{f}'$ (mid)	$\tilde{f}'$(right)
0	0	0	0	0	0	0
0.1	0.040599	0.040795	0.041333	0.031208	0.031429	0.032038
0.2	0.08021	0.080601	0.081678	0.062422	0.062864	0.064082
0.3	0.11886	0.119447	0.121064	0.093656	0.094319	0.096147
0.4	0.156594	0.157378	0.159537	0.124934	0.125819	0.128259
0.5	0.193473	0.194455	0.19716	0.156292	0.1574	0.160455
0.6	0.22957	0.230753	0.234011	0.187773	0.189106	0.192781
0.7	0.264973	0.266359	0.270179	0.219433	0.220993	0.225295
0.8	0.299781	0.301376	0.30577	0.251337	0.253128	0.258066
0.9	0.334107	0.335915	0.340899	0.283563	0.28559	0.291177
1	0.368074	0.370105	0.375699	0.316201	0.318469	0.324724
1.1	0.401823	0.404084	0.410314	0.349357	0.351874	0.358816
1.2	0.435505	0.438007	0.444906	0.38315	0.385925	0.393581
1.3	0.46929	0.472049	0.479654	0.417719	0.420765	0.429167
1.4	0.503369	0.5064	0.514758	0.453224	0.456554	0.465742
1.5	0.537953	0.541277	0.550445	0.48985	0.493481	0.503504
1.6	0.573284	0.576925	0.586972	0.527811	0.531764	0.542679
1.7	0.609637	0.613625	0.624633	0.567356	0.571657	0.583535
1.8	0.647329	0.6517	0.66377	0.608777	0.613456	0.626384
1.9	0.686733	0.691529	0.704781	0.65242	0.657515	0.671597
2	0.728284	0.733559	0.748141	0.698695	0.70425	0.719614
2.1	0.772505	0.778323	0.794416	0.748094	0.754165	0.770965
2.2	0.820025	0.826464	0.844292	0.801211	0.807865	0.826295
2.3	0.87161	0.878769	0.898609	0.85877	0.866092	0.886388
2.4	0.928207	0.936209	0.958409	0.921663	0.929758	0.952219

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2.5	0.991	1	1.025	0.991	1	1.025

Table 2

$\tilde{\beta}$=[0.25, 0.25, 0.25]				$\tilde{\beta}$=[0.50,0.50,0.50]		
η	$\tilde{f}'$(left)	$\tilde{f}'$(mid)	$\tilde{f}'$ (right)	$\tilde{f}'$(left)	$\tilde{f}'$(mid)	$\tilde{f}'$(right)
0	0	0	0	0	0	0
0.1	0.006021	0.006299	0.007068	-0.02008	-0.01977	-0.01891
0.2	0.014543	0.015099	0.016636	-0.03515	-0.03454	-0.03282
0.3	0.025565	0.026399	0.028705	-0.04523	-0.0443	-0.04174
0.4	0.03909	0.040203	0.043278	-0.05031	-0.04908	-0.04565
0.5	0.055121	0.056512	0.060357	-0.0504	-0.04886	-0.04458
0.6	0.073664	0.075334	0.07995	-0.04551	-0.04366	-0.03852
0.7	0.094729	0.096678	0.102069	-0.03565	-0.03349	-0.02749
0.8	0.118329	0.12056	0.126729	-0.02082	-0.01835	-0.01149
0.9	0.144487	0.147001	0.153953	-0.00105	0.001733	0.00946
1	0.173229	0.176029	0.183773	0.023652	0.02675	0.035347
1.1	0.204595	0.207685	0.216229	0.053272	0.056686	0.066161
1.2	0.238633	0.242018	0.251378	0.087795	0.091528	0.10189
1.3	0.275408	0.279093	0.289286	0.12721	0.131267	0.142528
1.4	0.315	0.318994	0.330043	0.17151	0.175897	0.188073
1.5	0.357509	0.361822	0.373755	0.220696	0.225419	0.238529
1.6	0.403061	0.407706	0.420559	0.274775	0.279844	0.293911
1.7	0.45181	0.456802	0.470619	0.333766	0.33919	0.354245
1.8	0.503944	0.509302	0.524138	0.397703	0.403495	0.419571
1.9	0.559692	0.565441	0.581363	0.466634	0.472809	0.48995
2	0.619336	0.625505	0.642594	0.54063	0.547207	0.565462
2.1	0.683217	0.689841	0.708199	0.619786	0.626785	0.646215
2.2	0.751752	0.758875	0.778625	0.704228	0.711676	0.732352
2.3	0.825451	0.833128	0.854423	0.794119	0.802045	0.824053
2.4	0.904942	0.913239	0.936269	0.889666	0.898107	0.921548
2.5	0.991	1	1.025	0.99113	1.000129	1.025125

Table 3

$\tilde{\beta}$=[1.00, 1.00, 1.00]				$\tilde{\beta}$=[1.25, 1.25, 1.25]		
η	$\tilde{f}'$(left)	$\tilde{f}'$(mid)	$\tilde{f}'$ (right)	$\tilde{f}'$(left)	$\tilde{f}'$ (mid)	$\tilde{f}'$ (right)
0	0	0	0	0	0	0
0.1	0.220318	0.219988	0.219065	0.170798	0.170394	0.169264
0.2	0.450355	0.449696	0.447852	0.35384	0.353034	0.350776
0.3	0.68936	0.688374	0.685618	0.548425	0.54722	0.543847
0.4	0.936024	0.934719	0.93107	0.753287	0.751694	0.747234
0.5	1.188432	1.18682	1.182312	0.966525	0.964561	0.959064
0.6	1.444003	1.442103	1.436791	1.185512	1.183207	1.176752
0.7	1.699419	1.69726	1.691222	1.406823	1.404218	1.396923
0.8	1.950549	1.94817	1.941518	1.626158	1.62331	1.615335
0.9	2.192349	2.189804	2.182684	1.838275	1.835262	1.82682
1	2.418736	2.416094	2.408699	2.036934	2.033852	2.025216
1.1	2.622422	2.619771	2.612348	2.214842	2.211813	2.20332
1.2	2.794684	2.792135	2.784996	2.363603	2.360774	2.352835
1.3	2.925033	2.922727	2.916263	2.473656	2.471203	2.464307
1.4	3.000736	2.998849	2.993553	2.534172	2.532302	2.527029

1.5	3.006065	3.004822	3.001317	2.532875	2.531832	2.528862
1.6	2.921144	2.920833	2.919923	2.455699	2.45577	2.455905
1.7	2.720085	2.721085	2.723827	2.286157	2.287686	2.291885
1.8	2.36793	2.370757	2.378585	2.004188	2.007599	2.01704
1.9	1.815553	1.820933	1.835867	1.58411	1.589947	1.606143
2	0.991	1	1.025	0.991	1	1.025

VI. CONCLUSIONS

The initial right value, left value and the mid values are taken from the given boundary conditions. As the velocity progresses from the initial point computations give the left and the right values which are respectively to the left and the right of the mid value as desired. So, from the above observations, we can conclude that the mid value of a triangular fuzzy number coincides with the crisp value of the original problem.

REFERENCES

1. *Goldstein, S. Proc. Camb. Phil. Soc. Vol. 35, pp. 388, 1939.*
2. *Mangler, W. ZAMM, Vol. 23, pp 241, pp.1943.*
3. *Falkner, V. M. and Skan, S. W. Phil. Mag. Vol. 12 pp 865, 1931.*
4. *Blasius, H. Z. Math. U. Phys., Vol. 19, pp. 1, 1908.*
5. *Prandtl, L. The Mechanics of Viscous Fluids, In W. F. Durand,Aerodynamics Theory III, 1935.*
6. *Goldstein, S. Proc. Camb. Phil. Soc., Vol. 26, pp 1, 1930*
7. *Goldstein, S. Modern Development of Fluid Dynamics, Vol. I, Oxford, 1938*
8. *Homann, F. ZAMM, Vol. 16, pp 153, 1936.*
9. *Froessling, N. Lunds Univ. Arrsskr, N. F. Avd. 2, Vol. 35, 1940.*
10. *Hiemenz, K. Thesis Gottingen and Dingl. Poly-Tech. J, Vol 326, pp 1911.*
11. *Hazarika, G. C. Shooting Method to Some Problems of Fluid Mechanics, Lambert Academic Publishing, 2014.*
12. *Roberts, S. M. and Shipman, J. S. Two point Boundary Value Problems-Shooting Methods, American Elsevier Co., New York, 1972.*
13. *Conte, S. D. Elementary Numerical Analysis – An Algorithmic Approach, Mc. Graw Hill Book Co. New York, 1965*
14. *Collatz, L. Numerical Treatment of Differential Equations, Springer Verlog, Inc., New York, 1966*
15. *Scarborough, J. B. Numerical Mathematical Analysis, Oxford and IBH Pub. Co., New Delhi, 1966.*
16. *Dutta, P., Boruah, H. and Ali, T.International Journal of Latest Trends in Computing, Vol. 2, Issue 1, pp. 99-107, 2011.*
17. *de Barros, L. C., Bassanezi, R. C. and Lodwick, W. A. A First Course in Fuzzy Logic, Fuzzy Dynamical Systems, and Biomathematics, Springer, 2017.*
18. *Chakravarty, S., Tapaswini, S. and Behera, D. Fuzzy Arbitrary Order System Fuzzy Fractional Differential Equations and Applications, Wiley, 2016.*
19. *Kaleva, O. Fuzzy Sets and Systems,Vol. 24, pp. 301-317, 1987.*
20. *Gomes, L. T., Barros, L. C. de and Bede, B. Fuzzy Differential Equations in Various Approaches, Springer, 2015.*
21. *Kermani, M. A. and Saburi, F. Applied Mathematical Sciences, Vol. 1, Issue 27, pp. 1299-1309, 2007.*
22. *Chalco-Cano, Y., Román-Flores, H. and Lodwick, W. A. A Note on Fuzzy Differential Equations. (This paper has been supported by Fondecyt-Chile through projects 1120665 and 1120674 and FAPESP-Brasil under the grant 2011/13985-0.)*
23. *Chetia, B. C. and Hazarika, G. C. Fuzzy Sets and Systems, Vol. 115, pp. 463- 469, 2000.*
24. *Mizukoshi, M. T., Barros, L. C., Chalco-Cano, Y., Román-Flores, H. and Bassanezi, R. C. Information Sciences, Vol. 177, pp. 3627-3635, 2007.*
25. *Ahmed, M. Z., Hasan, M. K. and Abbasbandy, S. The Scientific World Journal, pp. 1-11, 2013.*