A REVIEW ON ECG SIGNAL COMPONENT DECOMPOSITION

Jithin M* & Shayini R2

*1M.Tech Scholar, College of Engineering Thalassery
2Asst. Professor, College of Engineering Thalassery

ABSTRACT

It can be employed for multiple purposes, like denoising and features extraction, it is particularly suited for extracting electrocardiogram (ECG) wave-forms from ECG recordings. In the first paper, a nonlinear Bayesian filtering framework is proposed for the filtering of single channel noisy electrocardiogram (ECG) recordings. In the second paper presents efficient denoising and lossy compression schemes for electrocardiogram (ECG) signals based on a modified extended Kalman filter (EKF) structure. In the third paper, an extended nonlinear Bayesian filtering framework for extracting electrocardiograms (ECGs) from a single channel as encountered in the fetal ECG extraction from abdominal sensor.

Keywords - ECG, Denoising, Review

I. INTRODUCTION

The analysis of the electrocardiogram (ECG) is routinely performed to assess cardiac health status. Every ECG beat is composed of different waves, classically labeled as P, Q, R, S and T, which reflect, at the body surface, the electrophysiological activity of the heart. The cardiac cycle begins with the P wave, linked to atrial depolarization, followed by the QRS complex and T wave, which instead corresponds to ventricular depolarization and repolarization. Most of the clinically relevant information can be found within the amplitudes, shapes and intervals between these waveforms. Some examples are ST-waveform analysis for intrapartum fetal monitoring changes in P wave morphology due to various conditions, QT interval analysis and T wave alterations (TWA). So an accurate and robust procedure for automatic ECG labelling is an important goal for clinicians and biomedical engineers. A preliminary ECG components extraction phase, where the different waves are separated from each other, could surely simplify the task.

The extraction of high-resolution cardiac signals from a noisy electrocardiogram (ECG) remains an important problem for the biomedical engineering community the framework presented in first paper is based on a dynamic model, previously developed for the generation of synthetic PQRST complexes with their relationship to the beat-to-beat RR-interval timing. Considering the simplicity and flexibility of this model it is reasonable to assume that it can describe the dynamics of a broad class of normal and abnormal ECGs. Moreover, the applications of this model are not limited to ECGs and may be extended to other quasi-periodic signals. In recent works the authors employed this model to develop an Extended Kalman Filter (EKF) for noisy ECG filtering. In this paper, this synthetic ECG model is further modified and used in conjunction with several nonlinear Bayesian filtering approaches such as the EKF, Extended Kalman Smoother (EKS), and Unscented Kalman Filter (UKF). Furthermore, the model parameter selection is automated in order to adapt the method to different ECG channels. In order to evaluate the proposed method, different portions of white and colored Gaussian noises have been added to visually inspected clean ECGs recorded from various ECG lead configurations with differing morphologies. The Signal-to-Noise Ratios (SNRs) of the filter outputs have then been compared with several conventional ECG denoising schemes. An example of the filtering performance, in presence of real nonstationary muscle artifact (MA) is also presented. The results demonstrate that the proposed filters can accurately track the ECG signal in very low SNR conditions, where the cardiac signal is almost lost in background noise. This method is believed to serve as a novel framework for the model-based extraction of high-resolution ECG signals from noisy measurements.
In particular, since the presented method provides an accurate separation of nonlinear and nonstationary signal and in-band noise, it is hoped that this method will be suitable for applications such as the extraction of ECG late potentials from high resolution ECGs, or the noninvasive extraction of fetal ECG from the signals recorded from an array of electrodes placed on the maternal abdomen, where due to the low SNR of these signals, conventional filtering approaches do not give satisfactory results.

In second paper the KF framework has been further modified by adding 15 more equations to present the governing equations of the model parameters. In fact, the new proposed structure is aimed at estimating these new parameters, as well as the ECG signal. The added parameters are further used for reconstructing the ECG. This paper proposed algorithm puts into work both the denoising and compression approaches simultaneously, but based on a sequential representation on a beat-by-beat dynamical adaptive basis. Meanwhile, the model is nonlinear and requires the nonlinear counterparts of the conventional Kalman filter. Our proposed model-based framework is built upon an extended Kalman filter (EKF) structure.

In the last paper, we present an extended nonlinear Bayesian filtering framework for extracting electrocardiograms (ECGs) from a single channel as encountered in the fetal ECG extraction from abdominal sensor. The recorded signals are modeled as the summation of several ECGs. Each of them is described by a nonlinear dynamic model, previously presented for the generation of a highly realistic synthetic ECG. Consequently, each ECG has a corresponding term in this model and can thus be efficiently discriminated even if the waves overlap in time. The parameter sensitivity analysis for different values of noise level, amplitude, and heart rate ratios between fetal and maternal ECGs shows its effectiveness for a large set of values of these parameters.

II. LITERATURE REVIEW

A. A Nonlinear Bayesian Filtering Framework for ECG Denoising

In this paper, a nonlinear Bayesian filtering framework is proposed for the filtering of single channel noisy electrocardiogram (ECG) recordings [1]. The necessary dynamic models of the ECG are based on a modified nonlinear dynamic model, previously suggested for the generation of a highly realistic synthetic ECG. A modified version of this model is used in several Bayesian filters, including the Extended Kalman Filter, Extended Kalman Smoother, and Unscented Kalman Filter. An automatic parameter selection method is also introduced, to facilitate the adaptation of the model parameters to a vast variety of ECGs. This approach is evaluated on several normal ECGs, by artificially adding white and colored Gaussian noises to visually inspected clean ECG recordings, and studying the SNR and morphology of the filter outputs. The results of the study demonstrate superior results compared with conventional ECG denoising approaches such as bandpass filtering, adaptive filtering, and wavelet denoising, over a wide range of ECG SNRs.

The framework presented in this paper is based on a dynamic model, previously developed for the generation of synthetic PQRST complexes with their relationship to the beat-to-beat RR-interval timing. In recent works, the authors employed this model to develop an Extended Kalman Filter (EKF) for noisy ECG filtering. In this paper, this synthetic ECG model is further modified and used in conjunction with several nonlinear Bayesian filtering approaches such as the EKF, Extended Kalman Smoother (EKS), and Unscented Kalman Filter (UKF).

1. Propose Method

a. Modification of the Dynamic ECG Model

The dynamic model consists of a set of nonlinear dynamic state equations in the Cartesian coordinates:

\[
\begin{aligned}
\dot{x} &= \rho x - \omega y \\
\dot{y} &= \rho y + \omega x \\
\dot{z} &= -\sum_{i \in \{P,Q,R,S,T\}} \alpha_i \Delta \theta \exp \left( -\frac{\Delta \theta^2}{2\sigma_i^2} \right) - (z - \omega_0)
\end{aligned}
\]

(C)Global Journal Of Engineering Science And Researches
So in order to simplify the dimensions and later relate the model parameters with real ECG recordings, the ai terms in (5) will be replaced with:

\[ a_i = \frac{\alpha_i \omega}{\beta_i^2} \quad i \in \{P, Q, R, S, T\} \]

Where the \( \alpha_i \) are the peak amplitudes of the Gaussian functions used for modeling each of the ECG components, in millivolts. This definition may be verified from (5), by neglecting the baseline wander term \((z - z_0)\) and integrating the equation with respect to \(t\). With these changes, the new form of the dynamic equations in polar coordinates is as follows:

\[
\begin{cases}
\dot{r} = r(1 - r) \\
\dot{\theta} = \omega \\
\dot{z} = -\sum_{i \in \{P, Q, R, S, T\}} \frac{\alpha_i \omega}{\beta_i^2} \Delta \theta_i \exp \left( -\frac{\Delta \theta_i^2}{2\beta_i^2} \right) - (z - z_0)
\end{cases}
\]

(6)

Where \(r\) and \(\theta\) are, respectively, the radial and angular state variables in polar coordinates.

b. Observation Equations

The noisy ECG recordings are assumed to be observations for the KF. The relationship between the states and observations of the KF depends on the location of the electrodes and the origin of the measurement noise. For example, motion artifacts, environmental noise or bioelectrical artifacts such as EMG or electro gastric noise, may be assumed as the measurement noises. While the measurement noise can generally contaminate the ECG in a nonlinear and non-Gaussian form, the results of this paper are based on the assumption of additive Gaussian noise.

c. Estimation of the Model Parameters

Prior to the implementation of the filter, it is necessary to select the values of the process and measurement noise covariance matrices. Generally, by using \(m\) Gaussian kernels, the process noise vector has, leading to \((3m+2) \times (3m+2)\) a process noise covariance matrix, \(Q_k\). However, if the noise sources are assumed to be uncorrelated with each other, a reasonable approximation adopted here, then the matrix is simplified to be diagonal. The measurement noise covariance matrix is similarly considered to be diagonal.

The next step is to find an estimate for the covariance values of \(Q_k\). This may be done by using the error values as depicted in Fig. 1.

Fig.1 Average and standard deviation-bar of 30 ECG cycles of a noisy ECG.
Disadvantage
- The abnormal ECGs are not treated in this paper.
- The system needs some revisions are necessary in the filtering process to be able to simultaneously filter the normal and abnormal segments of the ECG.
- Another related issue is the study of the appropriateness of the filtering procedure.
- There is a problem concerning the convergence-time, stability, estimation bias, and preciseness of the filter results.

B. ECG Denoising and Compression Using a Modified Extended Kalman Filter Structure
This paper presents efficient denoising and lossy compression schemes for electrocardiogram (ECG) signals based on a modified extended Kalman filter (EKF) structure. We have used a previously introduced two-dimensional EKF structure and modified its governing equations to be extended to a 17-dimensional case. The new EKF structure is used not only for denoising, but also for compression, since it provides estimation for each of the new 15 model parameters. Using these specific parameters, the signal is reconstructed with regard to the dynamical equations of the model. The performances of the proposed method are evaluated using standard denoising and compression efficiency measures. For denoising, the SNR improvement criterion is used, while for compression, we have considered the compression ratio (CR), the percentage area difference (PAD), and the weighted diagnostic distortion (WDD) measure. Several Massachusetts Institute of Technology–Beth Israel Deaconess Medical Center (MIT–BIH) ECG databases are used for performance evaluation. Simulation results illustrate that both applications can contribute to and enhance the clinical ECG data denoising and compression performance. For denoising, an average SNR improvement of 10.16 dB was achieved, which is 1.8 dB more than the next benchmark methods such as MABWT or EKF2. For compression, the algorithm was extended to include more than five Gaussian kernels. Results show a typical average CR of 11.37:1 with WDD < 1.73%. Consequently, the proposed framework is suitable for a hybrid system that integrates these algorithmic approaches for clean ECG data storage or transmission scenarios with high output SNRs, high CRs, and low distortions.

1. Extended Kalman Filter Review
The EKF is a nonlinear extension of conventional Kalman filter that has been specifically developed for systems having nonlinear dynamic models. For a discrete nonlinear system with the state vector $x_k$ and observation vector $y_k$, the dynamic model and its linear approximation near a desired reference point may be formulated as follows:

$$\begin{align*}
\dot{x}_k &= f(x_k, u_k, k) \\
\approx f(\hat{x}_k, \hat{u}_k, k) + A_k (x_k - \hat{x}_k) + F_k (w_k - \hat{w}_k) \\
y_k &= g(x_k, u_k, k) \\
\approx g(\hat{x}_k, \hat{u}_k, k) + C_k (x_k - \hat{x}_k) + G_k (y_k - \hat{y}_k)
\end{align*}$$

(1)

where

$$\begin{align*}
A_k &= \left. \frac{\partial f(x, \hat{u}, k)}{\partial x} \right|_{x=\hat{x}} \\
F_k &= \left. \frac{\partial f(x, \hat{u}, k)}{\partial w} \right|_{x=\hat{x}} \\
C_k &= \left. \frac{\partial g(x, \hat{u}, k)}{\partial x} \right|_{x=\hat{x}} \\
G_k &= \left. \frac{\partial g(x, \hat{u}, k)}{\partial y} \right|_{x=\hat{x}}
\end{align*}$$

(2)

Here, $w_k$ and $v_k$ are the process and measurement noises, respectively, with covariance matrices $Q_k = E\{w_k w_k^T\}$ and $R_k = E\{v_k v_k^T\}$. In order to implement the EKF, the time propagation and the measurement propagation equations are summarized as follows:
2. Model-Based Denosing And Compression

Once the EKF structure is constructed, we can perform selected processing applications on ECG signals. The proposed framework can estimate any of its states according to the dynamical equations and the observations. In fact, they enable us to build up denoising and compression blocks, a concept that is addressed next.

a. Denoising

The proposed nonlinear Bayesian framework estimates its variables using the state dynamical equations and its observations, noisy phase $\phi$ and noisy ECG $s$. Since the ECG signal, $z$, is a state variable in the EKF structure, the filtering procedure provides its estimation, $\hat{x}_2$, which is regarded as the denoised version of the input signal.

b. Compression

A mathematical representation for the clean ECG signal can be obtained by integrating the last equation of the continuous form of EDM (1) with respect to $t$. This way, the ECG signal is formulated as a sum of five Gaussians as:

$$z(a_i, b_i, \theta_i) = \sum_{i\in\{P,Q,R,S,T\}} a_i \exp\left(\frac{-\Delta \theta_i^2}{2b_i^2}\right). \quad (11)$$

If we have only an estimated value for the $a_i$, $b_i$, and $\theta_i$, we can reconstruct the original ECG. Since we have considered these variables as the states of EKF, we can easily estimate their values from $\hat{x}_2$. But, the EKF updates its estimations when a new sample is observed. This means that the EKF estimates the $a_i$, $b_i$, and $\theta_i$ parameters time series, in a similar manner to $\theta$ and $z$. We expect a constant value for each of the 15 Gaussians' parameters during each heartbeat. This is especially true, because the amplitude, spread, and angular location of the PQRST do not vary within a single ECG beat. In practice, since the estimated series of $a_i$, $b_i$, and $\theta_i$ are not a definitely constant function, we use its average value over each heartbeat and use equation (12) to reconstruct the estimated ECG.

$$z_{rec} = \sum_{i=3,7} \hat{x}_i \exp\left(\frac{-[\hat{x}_i - \hat{x}_{i+10} \text{mod}(2\pi)]^2}{2\hat{x}_{i+5}^2}\right). \quad (12)$$

This way, for every detected beat of an ECG, we must store/transmit 15 values. Also, as it can be seen from (12) that we need to store/transmit the estimated phase $\hat{x}_2$ to be able to reconstruct the ECG. However, since the phase values are linearly distributed between $-\pi$ and $+\pi$, we can only encode the zero locations (i.e., the R-peaks of $\hat{x}_2$). In the decoder, we use these locations to assign the phase values between $-\pi$ and $+\pi$. A more accurate representation for reconstructing the compressed ECG through a sum of Gaussians is possible if we vary the number of Gaussian functions in (12). Clifford et al. proposed an extension of the EDM, which used an arbitrary number of Gaussians, with two Gaussians for each asymmetric turning point. As was shown using six–eight Gaussians is a more appropriate choice for the number of kernels used for reconstruction, but this would further affect the compression performance.

The proposal to use the EDM for compression was previously introduced an optimization scheme to find the LSE fit for the input ECG. This fit was mathematically optimal in the LSE sense, but did not use any dynamical adaptable information about the input ECG. In the previous approach, the nonlinear optimization has to be performed within each cycle of the
signal. Also, initial values of the parameters of the model are required. These initials together with the system dynamics enable us to find an optimal fit for the proceeding cycles through the recursive solution (3). The current implementation is also based on the EDM. However, our method uses the dynamical set of equations in the construction of an adaptive filter, which not only uses the ECG as an observation but also depends on the state dynamics. Furthermore, the EKF-based algorithm does not need to have the initial parameters for every cycle of the input signal. Hence, the proposed EKF-based technique is an efficient idea for ECG compression.

Disadvantage

- EDM will be distorted.
- The algorithm not more reliable

C. Fetal ECG Extraction by Extended State Kalman Filtering Based on Single-Channel Recordings

In this paper, we present an extended nonlinear Bayesian filtering framework for extracting electrocardiograms (ECGs) from a single channel as encountered in the fetal ECG extraction from abdominal sensor. The recorded signals are modeled as the summation of several ECGs. Each of them is described by a nonlinear dynamic model, previously presented for the generation of a highly realistic synthetic ECG. Consequently, each ECG has a corresponding term in this model and can thus be efficiently discriminated even if the waves overlap in time. The parameter sensitivity analysis for different values of noise level, amplitude, and heart rate ratios between fetal and maternal ECGs shows its effectiveness for a large set of values of these parameters. This framework is also validated on the extractions of fetal ECG from actual abdominal recordings, as well as of actual twin magnetocardiograms.

1. Proposed method

a. Extended Kalman Filter Framework for ECG Extraction

The goal of KF is to estimate the state of a discrete-time controlled process. Consider a state vector $x_{k+1}$ governed by a nonlinear stochastic difference equation with measurement vector $y_{k+1}$ at time instant $k + 1$:

$$
\begin{align*}
    x_{k+1} &= f(x_k, w_k, k + 1) \\
    y_{k+1} &= h(x_{k+1}, v_{k+1}, k + 1)
\end{align*}
$$

(1)

where the random variables $w_k$ and $v_k$ represent the process and measurement noises, with associated covariance matrices $Q_k = E\{w_kw^T_k\}$ and $R_k = E\{v_kv^T_k\}$. The extended Kalman filter (EKF) is an extension of the standard KF to nonlinear systems $f(\cdot)$ and $h(\cdot)$, which linearizes about the current mean and covariance. In order to improve the estimations, EKF can be followed by a backward recursive smoothing stage leading to the extended Kalman smoother (EKS). However, since EKS is a non causal method, it cannot be applied online but it is useful if a small lag in the processing is allowed.

In this study, a synthetic dynamic ECG model [25] is used to extract fECG from mixture of an mECG, one (or more) fECG(s), and other signals considered as noises. In polar coordinates, one ECG signal can be expressed as the sum of five Gaussian functions defined by their peak amplitude, width, and center, denoted $a_i$, $b_i$, and $\psi_i$, respectively:

$$
\text{ECG} = \sum_{i=1}^{5} a_i \exp\left(-\frac{(x-b_i)^2}{2b_i^2}\right) \cos(2\pi \psi_i x)
$$

The ECGs composing the observed mixture can be estimated by recursively applying the described EKF: at each step, one ECG is extracted according to a deflation procedure. In case of a mixture of fECG and one fECG, the first step extracts, from the raw recording, the dominant ECG (often the mECG) considering the concurrent ECG (respectively, fECG) and other noises as a unique Gaussian noise. After subtracting the dominant ECG from the original signal, the second step is the extraction of fECG from the residual signal. This procedure is referred to as sequential EKF or EKS (seq-EKF or seq-EKS). In this recursive extraction, during the first step, the concurrent ECG (i.e., fECG) and additional noise are modeled by Gaussian noises $v_k$ and $w_k$, which is not a very relevant assumption. In fact, although this assumption may be acceptable when there are not strong artifacts interfering with the ECG, it is no longer accurate when other ECG artifacts are considerable (i.e., at the first step) since the noise is no longer normally distributed. In addition,
concurrent ECGs can be confused with dominant ECG when their waves (especially QRS complexes) fully overlap in time. Meanwhile, resultant inaccuracies, which are generated by the previous steps of the ECG extraction, will propagate to the next steps while the residuals are computed.

b. Extension to Multiple ECGs: Extended State EKF

In this paper, the dynamic equations (2) and (3) are extended for simultaneously modeling $N$ ECGs mixed in a single observation. The related extended state vector $x_k = [\theta(1)_k, z(1)_k, \ldots, \theta(N)_k, z(N)_k]^T$ is thus defined by:

\[
\begin{align*}
\theta^{(1)}_{k+1} &= (\theta^{(1)}_k + \omega^{(1)} \delta) \mod(2\pi) \\
\theta^{(N)}_{k+1} &= (\theta^{(N)}_k + \omega^{(N)} \delta) \mod(2\pi) \\
\end{align*}
\]

\[
\begin{align*}
z^{(1)}_{k+1} &= -\sum_{i \in W_1} a^{(1)}_i \omega^{(1)} \delta \frac{\Delta \theta^{(1)}_{i,k}}{2b^{(1)}_i} \exp\left(-\frac{(\Delta \theta^{(1)}_{i,k})^2}{2b^{(1)}_i}\right) \\
+ z^{(1)}_k + \eta^{(1)}_k \\
\vdots \\
z^{(N)}_{k+1} &= -\sum_{i \in W_N} a^{(N)}_i \omega^{(N)} \delta \frac{\Delta \theta^{(N)}_{i,k}}{2b^{(N)}_i} \exp\left(-\frac{(\Delta \theta^{(N)}_{i,k})^2}{2b^{(N)}_i}\right) \\
+ z^{(N)}_k + \eta^{(N)}_k
\end{align*}
\]

Where each $[\theta(i)_k, z(i)_k]^T$ is related to one of the ECGs. Finally, the measurement process leads to express the measurement vector $y_{k+1} = [\phi(1)_{k+1}, \ldots, \phi(N)_{k+1}, s_{k+1}]^T$ as

\[
\begin{align*}
\phi^{(n)}_{k+1} &= \theta^{(n)}_k + v^{(n)}_{k+1}, \forall n \in \{1, \ldots, N\} \\
s_{k+1} &= \sum_{n=1}^{N} z^{(n)}_{k+1} + v^{(N)}_{k+1} \\
\end{align*}
\]

This extended state Kalman filtering procedure is referred to as parallel EKF or EKS (par-EKF, or par-EKS, respectively). par-EKS is more accurate to extract fECG from abdominal sensors than the seq-EKF or seq-EKF. Indeed, in the proposed method, all ECGs are jointly modeled by dynamic states so that only the state and measurement noise vectors are assumed to be normally distributed. Moreover, the extended state par-EKF fully models overlapping waves of several ECGs. Finally, the state and observation noises, $\eta_{nk}$ and $v_{nk}$, respectively, allow the filter to fit some variabilities of the ECG shapes. Although the model does not fit too large variations (for example, due to arrhythmia), an inspection of the residue will reveal these abnormal beats.

c. Model Parameters Estimation

The proposed par-EKF and par-EKS lie on several state parameters $\{a(n)i, b(n)i, \psi(n)i\} \in W_n, \forall n \in \{1, \ldots, N\}$. The procedure described below is an extension of the single ECG parameter estimation. The parameters estimation procedure first needs the R-peak detection for all ECGs to perform the time wrapping of the R–R intervals into $[0, 2\pi]$ to define

$\phi(n)_k$. The R-peaks are found from peak search in windows of length $T$, where $T$ corresponds to the R-peak period calculated from approximate ECG beat-rate. R-peaks with periods smaller than $7T/2$ or larger than $T$ are not detected. Although maternal R-peaks are easily detectable from the mixture, fetal R-peak detection is more complex due to its lower amplitude than mECG. Therefore, a rough estimation of fECG is obtained by using the seq-EKF algorithm, which now allows us to detect easily the fetal R-peaks. Then, for each ECG, each beat (defined by the signals between two
consecutive Rpeaks) is time wrapped into $[0, 2\pi)$.

The average of the ECG waveform is obtained by the mean of all time-wrapped beats, for all phases between 0 and $2\pi$. Finally, by using a nonlinear least-squares approach, the best estimate of the parameters in the minimum mean square error (MMSE) sense is found.

**Disadvantage**

- The proposed method fails to discriminate correctly when the two fMCGs when they overlap.
- Proposed method fail to apply on multichannel mixtures of mECG and fECG.
- This paper not applicable for synchronous echocardiography data.

**III. CONCLUSION**

In this paper, a signal decomposition model-based Bayesian filtering method (EKS6) has been introduced for ECG signal processing and separation into its components (P, Q, R, S and T waves), by employing an original dynamical model. In the proposed method, the ECG components are directly utilized as hidden state variables, and simultaneously estimated as a time series through an EKS. The simulation results demonstrated that EKS6 has the capability of correctly tracking ECG component waves, on a beat-to-beat basis. There are some theoretical advantages that EKS6 has over other recent works in this context. As compared with EKS2, that uses only two state variables, six state variables are employed, with the advantage of permitting ECG components separation (and not only ECG filtering). Compared to EKS4, it no longer depends on the amplitudes of the Gaussian kernels, so it is able to separate the ECG components, even when abrupt changes happens in the signal. Also, the matrix in EKS6 is not constant in time, hence making it able to better model the nuances in the ECG signal. Finally, EKS linearizes the dynamical system at an operating point by approximating the state model through a first order Taylor series approximation. The truncation of the Taylor series is a poor approximation for most non-linear functions. In fact, the accuracy of the linearization depends on the amount of local non-linearity in the functions being approximated. Then, the posterior mean and covariance estimations become suboptimal and model errors are introduced. This can lead to instability, particularly when the system dynamics are strongly nonlinear. The EDM proposed here for EKS6 was derived to reduced the nonlinearity of the state model with respect to previous solutions.

**REFERENCES**

